Geometric Design of Cooperative Spectrum Sensing for Cognitive Radios

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Abstract—Cooperative spectrum sensing is widely known to collect sensing information from cooperative sensors to assist cognitive radio transmitter to better judge the transmission opportunities, with appearance of hidden primary system terminals. However, it is usually ignored that a successful transmission can go through by requiring not only the spectrum availability at cognitive radio transmitter, but also spectrum availability at cognitive radio receiver. Using this special observation, we use Poisson distributed nodes and Boolean random network model to analyze the geometric region allowing CR transmission with the help of cooperative sensors. Contrary to intuition, we find that cooperative sensing is not always helpful and the region allowing CR transmission is no longer circular anymore. Using this geometric property, we further find that transmission link can remain bidirectional only under certain geometric conditions. Due to these observations, we develop a complete methodology so that a secondary transmitter can select helpful cooperative sensors corresponding to different receivers.

I. INTRODUCTION

Cognitive radio seems to be a possible solution for the problem of scarce radio spectrum resource in recent years [1]. In the overlay CR network [2], the most important part of cognitive radio is that the secondary users can successfully detect the existence of the primary users. In order to improve the performance of the spectrum sensing, many different techniques have been developed, such as: energy detector, cyclostationary detector, etc [3] to improve the performance of the detection. Nevertheless, due to path-loss or shadowing effect, the signal from primary user may be too weak that secondary users cannot detect the existence of primary users. In order to solve this problem, the concept of cooperative sensors has been proposed and many efforts show that it can really help improve the detection performance [4]–[7].

However, there are two myths about cooperative sensing, that is, more cooperative sensors is desirable and communication link can be bidirectional. To break these two myths, we first need to know that the successful transmission over a CR link relies on not only spectrum availability at transmitter side but also spectrum availability at receiver side [8]. Therefore, the function of cooperative sensor should be focused on retrieving the information at the receiver side instead of improving the performance at the transmitter like previous works. But the observation of cooperative sensors is not always relevant to the state of secondary transmitter and receiver. For example, the observation result of cooperative sensor at infinity must have nothing to do with the secondary transmission pair. Therefore, not all cooperative sensors can bring benefit for secondary users and the performance cannot be improved as increasing the number of cooperative sensors as shown in [4]. Because cooperative sensors need to feedback information to the transmitter, using cooperative sensors introduces additional communication overhead. Overhead may create significant loss of spectrum efficiency, especially with extremely large number of nodes [9]. If secondary user can distinguish which cooperative sensor can bring benefit for itself and which one not, it can turn off those non-beneficial cooperative sensors and save the overhead of the CR network. For example, [10] proposes a distributive algorithm to select cooperative sensors in infrastructure-based CR network.

For the second myth, bidirectional link is a common assumption in (Ad-Hoc) network. In generally, we may further assume that the fading statistics between transmission pair are the same and therefore the communication link is bidirectional. However, the state of transmitter and receiver side may be different, which results in the communication link being unidirectional [11], which is particularly true in opportunistic transmission. In [12], Tu and Chen first show that even though cooperative sensing can help recover the transmission link, it does not guarantee that this link is bidirectional even with pure path loss channel model in point source CR network. That is, we cannot say that communication is bidirectional by assuming symmetric property in physical layer. The reason is that the geometric shape of region allowing CR transmission is not a symmetric circle anymore in CR network.

In this research, we show that the similar phenomenon can be found in Ad-Hoc type CR network. By exploring the geometric property of CR transmission, we can find the region allowing secondary users to transmit is not circular even though the distribution of primary users is homogeneous. And the shape of the region depends on the location of the cooperative sensor. Such phenomenon also results in that the transmission link is not always bidirectional. From the view points of geometric property, only selecting specific cooperative sensor could maintain CR transmission bidirectional.

The rest of the paper is organized as followed: Section II describes the details of our model. In Section III, we describe what is general spectrum sensing model [8] and provide the definitions in this section. In Section IV, we provide the numerical result and points out when a $S_T$ needs to use $S_C$. We draw our conclusion in Section V.
II. SYSTEM MODEL

A. Network Geometry

To simplify the notations, we use $S_T$, $S_R$ and $S_C$ to denote secondary transmitter, receiver, and cooperative sensor respectively. We first consider a planar area where CR network of primary users and secondary users exist. Considering all the primary users are randomly distributed in the planar area, we assume that the spatial distribution of primary users follows homogeneous Poisson point process (PPP) $\Phi_p$ with density $\lambda_p$. Without losing generality, we assume that all of them are active in each time slot. All the secondary users are synchronous with primary users, and each of them senses the channel at the beginning of the time slot. In addition to sensing the channel by itself, $S_T$ can also choose some cooperative sensors nearby to help itself improve the performance of spectrum sensing.

B. Boolean Random Network Model

We use the Boolean model to describe the behaviors of transmitter and receiver. Notation $B(A, r)$ to denote circle region centered at $A$ with radius $r$ and $|B(A, r)|$ to be the area of region. As illustrated in Fig. 1, $B(S_T, r_d)$ is the interference region of secondary user depending on the transmission power of secondary transmitter. And $B(S_T, r_d)$ is the detection region, that is, secondary users can successfully detect any active transmitter if they are in this region. The region $B(S_R, r_p)$ is the interference protection region due to existence of primary transmitter, $S_R$ can successfully receive data if there is no primary transmitter in this region.

In traditional spectrum sensing, the $S_T$ may think that there is a communication link between $S_T$ and $S_R$ if they do not detect any primary transmitter. To simplify notation, we define indicator function $1(A, r)$ as:

$$1(A, r) = \begin{cases} 1, & \text{if no primary user in } B(A, r) \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

However, it is not the case in CR network. From Fig. 1, we can find that existence of opportunistic communication link depends on not only the transmitter but also receiver side. Here we use $r_d$ to denote the detection radius of $S_T$ and $r_p$ to denote the interference radius of primary transmitter. Then the opportunistic communication link exists if $1(S_T, r_d) = 1 \land 1(S_R, r_p) = 1$ and there is no opportunistic communication link if $1(S_T, r_d) = 0 \lor 1(S_R, r_p) = 0$. Therefore, the existence of communication link $1_{\text{link}}$ can be expressed as:

$$1_{\text{link}} = 1(S_T, r_d)1(S_R, r_p), \tag{2}$$

$1_{\text{link}} = 1$ if there exists communication link and $1_{\text{link}} = 0$ if there is no communication link.

III. GENERAL SPECTRUM SENSING MODEL

From now on, we can find that the opportunistic communication link can exist ($1_{\text{link}} = 1$) only if $1(S_T, r_d) = 1$. Therefore, we can focus our discussion on situation conditioned on $1(S_T, r_d) = 1$. Conditioned on $1(S_T, r_d) = 1$, $S_T$ needs to guess what state the receiver side is. Therefore, we formulate the problem as binary hypothesis test problem. Conditioned on $1(S_T, r_d) = 1$, the two hypotheses for opportunistic communication link are given by:

$$H_0 : \text{opportunity, } 1_{\text{link}} = 1(S_T, r_d)1(S_R, r_p) = 1$$
$$H_1 : \text{no opportunity, } 1_{\text{link}} = 1(S_T, r_d)1(S_R, r_p) = 0.$$

And a priori probability of $H_0$ and $H_1$ can expressed as:

$$P(H_0) = P(1(S_R, r_p) = 1|1(S_T, r_d) = 1) = \alpha$$
$$P(H_1) = P(1(S_R, r_p) = 0|1(S_T, r_d) = 1) = 1 - \alpha. \tag{3}$$

To design the optimal detection strategy, we consider the Bayesian criterion and the Bayesian risk is defined as:

$$R = P(H_0)P(\bar{H} = H_1|H_0) + wP(H_1)P(\bar{H} = H_0|H_1), \tag{4}$$

where $\bar{H}$ denote the prediction of hypothesis and $w$ represent the normalized cost for miss detection for existence of primary users (it also means that secondary system will pay much more cost for failed transmission). For Bayesian hypotheses test, the a priori probability is an important parameter for decision maker. Because the information at $S_R$ side is not available for $S_T$, $S_T$ cannot know the value of $\alpha$ previously. But it is reasonable assumption that $S_T$ can observe the link between $S_T$ and $S_R$ and learn this value. By observing $N$ times and account the times of $1(S_R, r_p) = 1$, $S_T$ can estimate $\alpha$. In fact, it is the problem of estimating the probability of succession/failures trial. Such problem can be solved by Laplace formula:

**Proposition 1:** The estimation of a priori probability $\alpha$ is:

$$\hat{\alpha} = \frac{n + 1}{N + 2}, \tag{5}$$

where $n$ is the number of times of $1(S_R, r_p) = 1$. 

Fig. 1. Illustration of interference model. A communication link can exist if primary users are not in the detection region $B(S_T, r_d)$ and interference protection region $B(S_R, r_p)$, that is, $1(S_T, r_d) = 1 \cap 1(S_R, r_p) = 1$. 


A. Without Cooperative Sensor

If there is no other cooperative sensor providing side information to \( S_T \), eq.(4) becomes the hypotheses test without any observation. In such case, the \( a \) priori probability \( \alpha \) and the cost \( w \) determine whether \( S_T \) should transmit data or not.

Proposition 2: Without cooperative sensor, \( S_T \) will transmit data only if \( \alpha \geq \frac{w}{w+1} \), that is:

\[
\hat{H} = \begin{cases} 
H_0, & \text{if } \alpha \geq \frac{w}{w+1} \\
H_1, & \text{if } \alpha < \frac{w}{w+1}.
\end{cases}
\]

Proof: The Bayesian risk of transmitting data (\( \hat{H} = H_0 \)) and keep silence (\( \hat{H} = H_1 \)) can be expressed as:

\[
R(H_0) = w(1-\alpha) \\
R(H_1) = \alpha.
\]

To obtain the smallest Bayesian risk, we have decision strategy:

\[
R(H_0) = w(1-\alpha) \frac{H_1}{H_0} \geq \alpha = R(H_1),
\]

that is, we hope that we can always select the decision with smallest risk. Rearranging the inequality, we finish the proof.

Actually, we can also think that the value of \( \alpha \) represents how reliable a communication is. The larger \( \alpha \) is, the communication link is more reliable. And \( \frac{w}{w+1} \) in eq.(6) can be interpreted as the minimum reliability a communication link should be. If the estimated value of \( \alpha \) is lower than \( \frac{w}{w+1} \), \( S_T \) will not transmit data even if \( S_T \) feel free for transmission because the risk of accessing channel is unacceptable.

B. With Cooperative Sensor

Considering that you need to make a decision with a supervisor. If you always follow the supervisor, for example, the supervisor says something is true and you regard it is true, when he says false and you also regard it is false. Because you always follow his advice, this supervisor must be necessary for you. On the other hand, if you always make decision by yourself no matter what the supervisor’s answer, then this supervisor is unnecessary for you.

The similar situation can be found in cooperative sensor in CR network. In some case, using cooperative sensor may not be a good strategy. For example, when the transmission pair has high value of \( \alpha \), it means that the communication link is reliability enough. \( S_T \) can ignore the information from cooperative sensor and always decides to transmit if \( 1(S_T, r_d) = 1 \). Then we can turn off the cooperative sensor to reduce the overhead of CR network in this case. On the other hand, if the cooperative sensor is small signal correlation, it is reasonable to get rid of this cooperative sensor because the information it provides is not relevant to the communication link.

In this section, we will discuss about when cooperative sensor can indeed minimize the Bayesian risk and when \( S_T \) can ignore the feedback information from cooperative sensor. We assume that there is a cooperative sensor \( S_C \) to transmit information to \( S_T \), and correlation between \( S_C \) and \( S_R \) is defined as:

\[
\mathbb{P}(1(S_C, r_d) = 1 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 1) = \beta \\
\mathbb{P}(1(S_C, r_d) = 0 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 0) = \gamma,
\]

where \( 1(S_C, r_d) \) is the feedback information from \( S_C \). From the viewpoint of hypothesis test, \( S_T \) would like to detect \( 1(S_R, r_p) \) with a \( a \) priori probability \( \alpha \) and observation \( 1(S_C, r_d) \).

In order to discuss about whether a \( S_T \) needs \( S_C \) or not, we provide some definitions here to simplify notation in the following discussion:

Definition 1: A cooperative sensor \( S_C \) is unnecessary if the \( S_T \) always decides to transmit data no matter what kind of information \( S_C \) transmits. That is, \( \hat{H} = H_0 \) if \( 1(S_C, r_d) = 1 \) or \( 1(S_C, r_d) = 0 \).

Definition 2: A cooperative sensor \( S_C \) is necessary if the \( S_T \) decides to transmit data or not depending on what kind of information \( S_C \) transmits. That is, \( \hat{H} = H_0 \) if \( 1(S_C, r_d) = 1 \) and \( \hat{H} = H_1 \) if \( 1(S_C, r_d) = 0 \).

Definition 3: A cooperative sensor \( S_C \) is useless if the \( S_T \) always decides not to transmit data no matter what kind of information \( S_C \) transmits. That is, \( \hat{H} = H_1 \) if \( 1(S_C, r_d) = 1 \) or \( 1(S_C, r_d) = 0 \).

Definition 4: A transmission allowable region is the place where the probability of transmission of \( S_T \) is not 0. That is the region where \( \mathbb{P}(\hat{H} = H_0) \neq 0 \).

According to these definitions, we can obtain the following proposition regarding sensor selection.

Proposition 3: Cooperative sensor is unnecessary if \( \alpha > \frac{w}{w+\gamma} \), is necessary if \( \frac{w(1-\alpha)}{w+\gamma} < \alpha < \frac{w}{w+\gamma} \), and is useless if \( \frac{w(1-\gamma)}{w+\gamma} < \frac{w(1-\gamma)}{w+\gamma} \).

Proof: We first prove that the unnecessary condition for \( S_C \) and the other two proofs can follow the same way. The best strategy for decision maker under Bayesian criterion is likelihood ratio test [13] and because \( S_T \) always decides to transmit (\( \hat{H} = H_0 \)) no matter what information \( S_C \) feedbacks. That is, \( S_T \) always regard primary user not existing.

\[
\hat{H} = \begin{cases} 
H_0, & \text{if } 1(S_C, r_d) = 1 \text{ or } 1(S_C, r_d) = 0 \\
H_1, & X.
\end{cases}
\]

According to [13], then we can get following inequality:

\[
\begin{align*}
\mathbb{P}(1(S_C, r_d) = 1 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 1) &= \frac{1-\gamma}{\beta} < \frac{\alpha}{w(1-\alpha)} \\
\mathbb{P}(1(S_C, r_d) = 0 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 0) &= 1 - \frac{1-\gamma}{\beta} < \frac{\alpha}{w(1-\alpha)}
\end{align*}
\]

Rearranging the inequality and we finish the first proof.
According to the same logics, the decision strategy of $S_T$ if $S_C$ is necessary and useless is:

**necessary condition:** (i.e. $S_T$ makes decision according to $1(S_C, r_d)$)

\[
\hat{H} = \begin{cases} 
H_0, & \text{if } 1(S_C, r_d) = 1 \\
H_1, & \text{if } 1(S_C, r_d) = 0.
\end{cases}
\]  

(10)

**useless condition:** (i.e. $S_T$ always regard primary user existing)

\[
\hat{H} = \begin{cases} 
H_0, & X \\
H_1, & \text{if } 1(S_C, r_d) = 1 \text{ or } 1(S_C, r_d) = 0.
\end{cases}
\]  

(11)

Derive the corresponding likelihood ratio test by following the similar way in eq.(8) and eq.(9) and we finish the proof.

C. Interpretation of Communication Region

Because the value of $\alpha$ depends on the location of $S_R$, we can divide the transmission allowable area around $S_T$ into unnecessary and necessary region according to the location of $S_R$. And the region out of transmission allowable region, we call useless region.

$S_T$ has minimal transmission allowable region if $S_T$ has no cooperative sensor $S_C$. If $S_R$ is in this minimal transmission allowable region ($\alpha \geq \frac{w^2}{w^2+d^2}$), $S_T$ can transmit data when $1(S_T, r_d) = 1$ without the help of $S_C$. Once $S_T$ has a $S_C$, we can further divide the transmission allowable into unnecessary and necessary region. If $S_R$ is near $S_T$ enough to be in the unnecessary region ($\alpha \geq \frac{w^2}{w^2+d^2}$), $S_T$ can always transmit data if its own detection result is $1(S_T, r_d) = 1$ (no primary users) no matter what information $S_C$ feedbacks. Because the decision result of $S_T$ do not depend on the feedback information from $S_C$, we call the $S_C$ unnecessary. The unnecessary region always locates in the minimal transmission allowable region. The reason is that the minimal transmission allowable region is also the place where $S_R$ is close enough to $S_T$. In such case, the communication link is reliable enough even without the help of $S_C$.

If $S_R$ moves away from $S_T$, then it enters into the necessary region ($\frac{w^2(1+\gamma)}{w^2+d^2} \leq \alpha \leq \frac{w^2}{w^2+d^2}$), where $S_T$ decides to transmit data or not depending on the feedback information from $S_C$. Only while feedback information from $S_C$ is $1(S_c, r_d) = 1$, the $S_T$ can transmit the data. Here we say necessary does not mean that $S_T$ can transmit data only with the help of $S_C$. For example, $S_T$ may be in the minimal transmission allowable region but is out of the unnecessary region. In this case, if $S_R$ is in necessary region, $S_T$ still can transmit data without the help of $S_C$. But $S_C$ can help the $S_T$ to reduce the expected Bayesian risk of transmission. If $S_R$ move further out of transmission allowable region without the help of $S_C$, $S_T$ can transmit data only while feedback information from $1(S_c, r_d) = 1$. In this circumstance, $S_C$ is really necessary to $S_T$.

If $S_R$ is in the useless region, $S_T$ always cannot transmit data to $S_R$ even with the help of cooperative sensor. Because $S_T$ always decides not to transmit data no matter what kind of information $S_C$ feedbacks, therefore the probability of transmission is 0. In such case, $S_T$ needs to choose other cooperative sensor to recover the useless region.

IV. EXPERIMENT

In section III, we provide the condition that when cooperative sensor can bring benefit for $S_T$ from the view point of link layer. In this section, we will show how to employ the result of previous section to know whether $S_T$ needs to use cooperative sensor or not and how cooperative sensor helps to increase the communication region of $S_T$ in geographic viewpoint.

A. Impact of Primary Users Network

At first, we discuss about the value of $\alpha$, $\beta$ and $\gamma$ in the Boolean random network model. Because of homogeneous distribution of all the primary users, according [14], the number of primary users in a region $B(A, r)$ follows Poisson distribution with mean $\lambda_p |B(A, r)|$. We use the notation $B_c(d_A, B, r_1, r_2)$ to denote the common region of two circles $A, B$ with distance $d_A, B$ and radius $r_1, r_2$ respectively (Fig. 2). The value of $\alpha$ is actually the probability that there is no primary users in the region $B(S_R, r_p) – B_c(d_{T,R}, r_d, r_p)$, where $d_{T,R}$ is the distance between $S_T$ and $S_R$ respectively (and we use $d_{T,C}, d_{R,C}$ to denote the distance between $S_T – S_C$ and $S_R – S_C$ in the following). In this way, we can express $\alpha$ as follows:

\[
\alpha = P(1(S_R, r_p) = 1 | 1(S_T, r_d) = 1) = e^{-\lambda_p \pi r_p^2} e^{\lambda_p |B_c(d_T, r_d, r_p)|}.
\]  

(12)

For $\beta$, it is the probability of no primary users in the region $B(S_C, r_d)$ conditioned on no primary users in $B(S_T, r_d)$ and $B(S_R, r_p)$. And $\gamma$ is the probability of at least one primary users in the region $B(S_C, r_d)$ conditioned on primary users in $B(S_T, r_d)$ but at least one in $B(S_R, r_p)$. Therefore, we can express $\beta$ and $\gamma$ respectively as follows:

\[
\beta = P(1(S_C, r_d) = 1 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 1) = e^{-\lambda_p [B(S_C, r_d) \cup B_c(d_T, r_d, r_p) \cup B_c(d_R, r_d, r_p)]} 
\]  

(13)

\[
\gamma = P(1(S_C, r_d) = 0 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 0) = 1 - P(1(S_C, r_d) = 1 | 1(S_T, r_d) = 1, 1(S_R, r_p) = 0) = e^{\lambda_p [B_c(d_T, r_d, r_p)]} - e^{\lambda_p |B(S_T, r_d) \cup B_c(d_T, r_d, r_p)|} 
\]  

(14)
With the statistical information $\alpha$, $\beta$ and $\gamma$, $S_T$ can use the Proposition 3 to determine whether to use feedback information from $S_C$.

B. Numerical Result

In the simulation, we use the parameter setting as following: $r_d = 10$, $r_p = 8$, $\lambda_p = 2.5 \times 10^{-3}/m^2$, $w = 9$, $S_T = (0, 0)$. We first discuss the transmission allowable region under the case with and without the help of $S_C$. And then we will show that the communication link in CR may not be the bidirectional in Fig. 5.

In Fig. 3 and Fig. 4, $S_C$ is located at (2, 0) and (5, 0) respectively and $S_T$ is located at (0, 0) in both case. We can find that the transmission allowable region without the help of $S_C$ is a circle due to homogeneous distribution of primary users. If the $S_R$ is in this minimal transmission allowable region, the detection result of $S_T$ can almost represent the state of $S_R$ ($\alpha > \frac{w}{w+1}$). In such case, $S_T$ can transmit data according to its own detection result. As we discussed in Section. III-C, we can observe that the unnecessary region is always inside the minimum transmission allowable region. It is theoretically due to the homogeneous property of distribution of primary users, the correlation between $\beta$ and $\gamma$ to be positive, that is, $\beta + \gamma \geq 1$. In this way, we can find that $\frac{w\gamma}{\sqrt{\beta w+\gamma}} > \frac{w}{w+1}$. Therefore, $\alpha > \frac{w\gamma}{\sqrt{\beta w+\gamma}}$ (in unnecessary region) also means that $\alpha > \frac{w}{w+1}$ (in minimal transmission allowable region without $S_C$). And the necessary region is between unnecessary region (thin solid line) and transmission allowable region with $S_C$ (dashed line). On the other hand, because $S_C$ cannot provide the relevant information about $S_R$ if it is too far away from $S_R$, the unnecessary region is always located at opposite side of the $S_C$. The similar phenomenon can also be found in transmission allowable region with $S_C$. The additional transmission allowable region is located at the side of $S_C$. The primary users at the left side of $S_T$ is still hidden to $S_T$ without the help of $S_C$. If $S_C$ moves away from $S_T$, as illustrated in Fig. 4, we can find that the unnecessary and transmission allowable region is enlarged. Because the additional transmission allowable region depends on the location of $S_C$, it means that $S_T$ needs to select different $S_C$ according to different $S_R$ when we are designing our network functions of CR network.

Another interesting issue is whether a communication link is bidirectional or not in CR network. In Fig. 3 and Fig. 4, we show that the transmission allowable region is not a circle anymore even for a point source. If a $S_R$ located in the minimal transmission allowable region of $S_T$, then the communication link is bidirectional. It is the reason that minimal transmission allowable is circular, therefore, $S_T$ is located in the minimal transmission allowable region of $S_R$ if $S_R$ is also located in that of $S_T$. However, it is not the case if $S_R$ is located out of minimal transmission allowable region of $S_T$. In this case, $S_T$ can communicate with $S_R$ only with the help of cooperative sensor. But the $S_C$ may not help $S_R$ to recover the communication link (here we assume that $S_T$ and $S_R$ use the same cooperative sensor to reduce the overhead of CR network) from $S_R$ to $S_T$. Here we present an example that the $S_C$ may not be helpful to keep a communication link to be bidirectional. In Fig. 5(a), we assume that $S_T$ and $S_R$ is located at (0, 0) and (6, 0) respectively and they have a common $S_C$ which is able to keep bidirectional communication link. The solid line is the region of $S_T$ where $S_C$ helps $S_T$ to communicate with $S_R$ and dashed line is that of $S_R$. We can observe that only the cross region in Fig. 5(a) can keep communication between $S_T$ and $S_R$ be bidirectional. Fig. 5(b)–(d) illustrate the cross region under different locations of $S_R$. We can observe that area of cross region decreases when the $S_R$ moves away from $S_T$. Therefore, there is a tradeoff between transmission distance and the bidirectional property of communication link. In other words, we can shorten our transmission distance for each hop to keep bidirectional property or we can use longer transmission link but losing bidirectional property.

Finally, we summarize the steps of cooperative sensor
Sensing Algorithm is as follows:

**Step 1:** At the meanwhile, \( S_C \) use Laplace formula to learn the value of \( \alpha \) between \( S_R \) and \( S_C \).

**Step 2:** At the simultaneously, \( S_C \) feedbacks information to \( S_T \) and \( S_T \) uses these information to learn \( \beta \) and \( \gamma \).

**Step 3:** By applying Proposition 3, \( S_T \) can determine whether to use cooperative sensor or not.

### V. Conclusion

For a long time, cooperative sensors are regarded as solution to solve the problem of hidden terminal problem in CR network. However, it also introduces other problems in addition to overhead of cooperative sensing information. In this paper, we use the concept of general spectrum sensing to model the relationship between \( S_T \), \( S_R \) and \( S_C \). The benefit of this model is that secondary users can only consider the information about the link level and using statistical inference to obtain the condition of the communication link. By using Laplace formula, \( S_T \) can learn the relationship among \( S_T \), \( S_R \) and \( S_C \), and use these information to determine whether to use cooperative sensor or not. With this geometric model, we suggest the conclusions for cooperative spectrum sensing:

1. The cooperative sensors are not always helpful to recover the CR communication link and the transmission region is no longer circular even though the distribution of primary users is homogeneous (symmetric property has been destroyed).

2. Cooperative sensing can recover some CR transmission links but it does not guarantee that this link is bidirectional. Only through selecting specific cooperative sensor could the CR transmission link be bidirectional.

3. Proposition 4 as the complete algorithm of cooperative spectrum sensing.

### References


