Synthetic Aperture Radar Construction of Spectrum Map for Cognitive Radio Networking

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ABSTRACT
To network cognitive radios that require spectrum sensing to identify transmission opportunities emerges as a critical technology to facilitate spectrum efficient wireless communications. We propose to apply synthetic aperture radar technology to construct spectrum map so that efficient cognitive radio networking operation is possible in this paper. We demonstrate such an approach indeed realizing spectrum map.

Categories and Subject Descriptors
C.2.1 [Computer-communication Networks]: Network Architecture and Design—Wireless communication

General Terms
Design

Keywords

1. INTRODUCTION
After the first decade of technology development, cognitive radio (CR) or dynamic spectrum access has been considered as the most promising technology to facilitate spectrum efficiency [12, 8]. To realize the concept of cognitive radio, spectrum sensing to identify spectrum hole or white space of the primary system, such that the secondary cognitive radio can opportunistically transmit using this opportunity, while tremendous amount of literatures addressed this issue. However, due to hidden terminal problem and fading, it is not trivial to reliably obtain the information of spectrum hole or white space. Cooperative sensing [7, 18, 13] is therefore developed to recent good attention in research. However, up to this point, cognitive radio has been considered as an opportunistic transmission from the CR transmitter to CR receiver, and as an opportunistic links. In practice, if we really want to implement CR technology, we must be able to enable multi-hop CR transportation from source node to destination node, as a general sense of cognitive radio network (CRN) with the help of cooperative relay through all nodes which may possibly involve [4, 3]. An immediate challenge would be to proceed routing over such general CRN [3] or ad hoc CRN [9] without distinguishing primary system and secondary system anymore. When we route packets over CRN, power control is another critical issue to ensure CR concept working through the entire CRN [15]. All these situations suggest that traditional spectrum sensing to detect spectrum hole or white space at the link level is not enough.

Some efforts have been conducted by introducing more technologies into spectrum sensing, such as knowledge-based reasoning [19] or statistical inference [5]. An interesting technique, cognitive radio network tomography, has been introduced [21] by similar approach like medical tomography or Internet tomography, to send a test packet into CRN so that more information not only at the link level but also at the network level can be collected, including crucial radio resource availability to serve the requirements in [9, 15, 6, 2]. Traditional spectrum sensing can be considered as a class of CRN tomography. The network level information still treats multiple-link operation in CRN. In [17], a different view has been raised. The availability of a CR link is actually not determined by the availability of CR transmitter end only, while almost all current spectrum sensing papers assume, but also the availability of CR receiver end. We therefore conclude that spectrum availability for CRN operation requires simultaneous knowledge of both radio resource and location. In other words, instead of spectrum sensing, we actually need the information to construct a spectrum map (i.e., x, y, and radio resource) to successfully execute CRN functions such as routing and power control.

The major contribution of this paper is to provide a scheme for CRN when constructing the spectrum map by utilizing radio remote sensing technology, which can be achieved by only few sensors. Moreover, the spectrum map constructed by the proposed scheme provides not only knowledge of radio resource but also accurate location information of active sources.

2. RELATED WORKS
Motivated by [20] to establish radio imaging by wireless
networking, we reciprocally implement spectrum map by radio remote sensing technology, while the most well known might be synthetic aperture radar (SAR) [14, 1, 10, 11]. In 1962, the principle of synthetic aperture imaging applies earth rotation was first proposed by Martin Ryle. Martin Ryle and Tony Hewish jointly won the Nobel Prize for Physics in 1974 for their work on the establishment of radio aperture synthesis and its role in the discovery of pulsars. For all technology of wireless communications nowadays, signals are transmitted through wireless channel by electromagnetic waves and thus there is intensity of electric field for modulated signals. According to the physical facts, an electric filed is similar to Newton’s gravitational field that contains the concept of position information implicitly. With the inspiration of their work, we resort to the concept that calculates spatial coherence of electric fields from the same active source to construct the spectrum map.

In 2005, Haykin [8] proposed a space-time processing technique by assigning numerous sensors in the radio frequency environment to measure and probe the scene of radio resource in cognitive radios. However, if number of sensors are limited and some active users are far away from neighboring sensors, the position of active users can be inaccurate. How many sensors are sufficient is an issue and generally sparse property should be concerned. The fundamental reason is that a sensor can operate detection or spectrum estimation but a sensor contains no position information from active sources. Hence, this becomes a shortage for the establishment of the spectrum map. In contract to casting a number of sensors, the proposed scheme is similar to radio remote sensing. We provide an another solution to construct spectrum map by utilizing the deterministic property of electric field with SAR technology.

The remainder of the paper is organized as follows. In section 3, the mathematical model and design of the proposed solution for construction of the spectrum map are formulated. In section 4, numerical simulations are evaluated. Finally, this paper is concluded in section 5.

3. SPECTRUM MAP

In the following, instead of choosing (x, y, z) for the expression of position in Cartesian coordinate, we use (l, m, w) and (u, v, w) for the consistency with related literatures [11].

3.1 System Model

Given a Cartesian coordinate $C_I$ and $(l, m, w)$ denotes its coordinate location and another Cartesian coordinate $C_V$ and $(u, v, w)$ denotes its coordinate location. Besides, there are two specific square areas named $(l, m)$-plane (or, $Ω_l$) which is on $C_I$ and $(u, v)$-plane (or, $Ω_V$) which is on $C_V$. These two planes are shown in Fig. 1. Firstly, $(l, m)$-plane is composed by $M_I$-by-$M_I$ small grids. Each grid has $dΩ_I$ unit area. Secondly, the other area $(u, v)$-plane is composed by $M_V$-by-$M_V$ small grids and each grid has $dΩ_V$ unit area. Generally, $M_V < M_I$.

For a general wireless communication scenario, there are many active sources composed by $dA_U$ number of active users and $dA_S$ number of base stations in the $(l, m)$-plane. Fig. 2 shows a simplified illustration. These active users can be either primary users or secondary users. Moreover, in an array of $n_a$ antennas, these antennas are settled on the u-axis in $(u, v)$-plane and position information of each antennas are known.

Spectrum map $I_f(l,m)$ can be constructed by applying the technique of radio interferometry [11] from an array of $n_a$ antennas on the $(u, v)$-plane. $I_f(l,m)$ provides the intensity information of transmitting signals at carrier frequency $f$ of active sources and their corresponding information of coordinate position $(l, m)$ on the $(l, m)$-plane.

**Assumption 1**: We assume the distance (in Fig. 2) between active sources and antennas are far away. That is, active sources are seen as point sources for sensors and antennas are coherent.

![Figure 1: Illustration of the relation between the (u, v)-plane and the (l, m)-plane.](image1)

![Figure 2: Illustration of the sensor device.](image2)
sphere has area $S$.

**Assumption 2:** In (1), we ignore that the electric field is a vector quantity. That is, we ignore polarization. In addition, we assume that the space inside the observed sphere is empty.

**Assumption 3:** We assume radiation from different locations on the observed sphere is spatially uncorrelated. That is, $\langle E_i(q)E_j^*(q) \rangle = 0$, where $\langle \cdot \rangle$ represents the spatial coherence.

**Definition 1:** Spectrum map, $I_f(s)$, or image is the active source intensity in the direction of unit-length vectors $s = \frac{r}{|r|}$ on a observed sphere and it is $I_f(s) \equiv \langle \epsilon_f(s) \rangle$.

**Definition 2:** Visibility, $V_f(r_1, r_2)$, is the measured spatial correlation (coherence) of the electric fields between two antennas $i$ and $j$ with locations $r_1$ and $r_2$ on $\Omega_f$, respectively.

$$V_f(r_1, r_2) = \langle E_i(r_1)E_j^*(r_2) \rangle = \int I_f(s)e^{-2\pi i \lambda s^T(r_1-r_2)}dA,$$  

(2)

where $|s|^2dA = d\Omega_f$ and $*$ denotes complex conjugate.

Besides, the vector $\varsigma = r_2 - r_1$ is call a baseline between two antennas. In an array of $n_a$ antennas, there are $\frac{n_a(n_a-1)}{2}$ pairs of combinations can be established. That is, these combinations can produce $\frac{n_a(n_a-1)}{2}$ baselines.

### 3.3 Spectrum Map in Cartesian Coordinate

Since the formula in (2) is represented in spherical coordinate, in order to give a more intuitive point of view, spectrum map in Cartesian coordinate is introduced. To consider a planar array, the baselines can be formed as $\varsigma = r_2 - r_1 = \lambda [u, v, w = 0]^T$ and $s = [l, m, n = \sqrt{-l^2-m^2}]^T$, where $\lambda$ is the wavelength of an electromagnetic wave with frequency $f$ and $T$ denotes matrix transpose. We also have $dA = \frac{dudv}{\sqrt{l^2-m^2}}$, where the factor $\sqrt{l^2-m^2}$ is the Jacobian for the transformation from spherical to rectangular coordinates. Therefore, (2) becomes

$$V_f(u, v) = \int \int V_f(l, m)e^{-2\pi i j(u l + v m + c)}dldm \sqrt{l^2-m^2}.$$  

(3)

By Assumption 1, $|l^2+m^2|$ is small, the factor $\sqrt{l^2-m^2}$ is neglected and from (3) we have

$$I_f(l, m) = \int \int V_f(u, v)e^{2\pi i j(u l + v m + c)}dudv.$$  

(4)

This form of the spectrum map is only valid when the baselines of a synthesis array are coplanar. Moreover, the relationship between the spectrum map and visibility is a Fourier transformation pair, which is $I_f(l, m) \overset{F^{-1}}{\mapsto} V_f(u, v)$, where $F \{ \}$ denotes Fourier transformation.

**Definition 3:** Dirty beam, $B(l, m) = F^{-1} \{ S(u, v) \}$, is a point spread function, where $S(u, v)$ is a sampling function which samples values of the visibility. That is, $S(u, v)$ equals to one for each measured $(u, v)$ pair and otherwise equals to zero.

**Definition 4:** Dirty image, $I_{D, f}(l, m)$, or a dirty spectrum map is

$$I_{D, f}(l, m) = \int \int V_f(u, v)S(u, v)e^{2\pi i j(u l + v m + c)}dudv = F^{-1} \{ V_f(u, v) \} \otimes F^{-1} \{ S(u, v) \},$$  

(5)

where $\otimes$ is a convolution operator.

By (5), a dirty spectrum map can be briefly obtained by

$$I_{D, f}(l, m) = I_f(l, m) \otimes B(l, m).$$  

(6)

### 3.4 General Scenario in Wireless Communications

Assume there are totally $D$ active sources in communica- tions, where $D = d_{AV} + d_{BS}$. The spectrum map is composed by intensities of multiple active sources when transmitting power,

$$I_f(l, m) = \sum_{d=1}^{D} I_f(l, m) \delta(l - l_d) \delta(m - m_d).$$  

(7)

and the corresponding visibility at an instantaneous observation time $k$ is

$$V_f(u^k, v^k) = \sum_{d=1}^{D} I_f(l, m) e^{-2\pi i j(u^k l_d + v^k m_d)}.$$  

(8)

Since there are $n_a$ antennas on $\Omega_f$, we formulate the visibility which is obtained by pairs of antennas into a matrix form. By selecting a reference point $[11] (u_0^k, v_0^k)$ on $\Omega_f$, $u^k = u_0^k - u_0^k$ and $v^k = v_0^k - v_0^k$, (8) becomes a correlation (coherence) matrix $R^k$ at time $k$ and frequency $f$, which is $R^k = A_kB^{'T}_k$, where

$$a_k = [a_k(l, m_1), \ldots, a_k(l, m_D)],$$  

$$A_k = \begin{bmatrix} a_k(l, m_1), & \cdots, & a_k(l, m_D) \end{bmatrix}^T,$$  

$$B = \text{diag}(I_f(l, m_1), \cdots, I_f(l, m_D)).$$

“diag” denotes a diagonal matrix and $\mathbb{H}$ stands for conjugate and transpose. Generally, the dirty image can be obtained by averaging observed visibility during a long period $\Delta t$ of observations,

$$I_{D, f}(l, m) = \frac{1}{\Delta t} \sum_{k=0}^{\Delta t} a_k(l, m)R^k a_k(l, m).$$  

(10)

### 4. NUMERICAL RESULTS

<table>
<thead>
<tr>
<th>User 1</th>
<th>User 2</th>
<th>User 3</th>
<th>User 4</th>
<th>User 5</th>
</tr>
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<td>152</td>
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<td>98</td>
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<tr>
<td>217</td>
<td>236</td>
<td>281</td>
<td>135</td>
<td>375</td>
</tr>
</tbody>
</table>

Table 1: Position information of 10 active users on the (l, m)-plane.

In this section, numerical experiments are established. Given that $d_{AV} = 10$ (10 active users) in the (l, m)-plane with the same carrier frequency $f = 3$ GHz. Consequently, the corresponding wavelength is $\lambda = \frac{c}{f} = 0.1(\text{meter})$. In Fig. 4(a) and Fig. 4(b), $n_a = 4$ antennas are settled in the (u, v)-plane and hence we have $\frac{n_a(n_a-1)}{2} = 6$ baselines. For
the linear array of antennas, the non-redundant array strategy [11] is applied for the placement of antennas. In the (u, v)-plane, the unit is the same as the wavelength, i.e., \( \lambda = 0.1 \) (meter). Therefore, we show that in Fig. 4(a), at least 3 meters spacing is required for the placement of antennas on the (u, v)-plane. Furthermore, the size of the (l, m)-plane is formed by 500-by-500 grids \((M_l = 500)\) and the absolute intensity of transmitting power of all 10 active users equals to \(10^6\). The position of active users are fixed and their corresponding positions are shown in Table 1. The free space path loss model is also applied for the active sources and the pass loss with distance \(d\) is based on

\[
P_r = \frac{P_t \cdot \sqrt{G_t \cdot G_b}}{4\pi d^2},
\]

where \(P_r\) and \(P_t\) denote the received power and the transmitted power, respectively. \(\sqrt{G_t G_b}\) is the product of the transmit and receive antenna field radiation patterns in the line-of-sight direction. We choose \(G_t = 1\) for omnidirectional antennas. Besides, baselines on the (u, v)-plane are established through a period of observations. In our case, 63 baseline sets are obtained after numerous times of observations, which corresponds to the rotation of the linear arrays of antennas. That is, in Fig. 2, the observation time \(H_0\) is from \(-\pi_i\) to \(\pi_i\) with 0.1 (rad) shift for each observation. In addition, the declination angle \(\delta = 40^\circ\) is chosen (Fig. 1). In simulations, the arithmetic of rounding is applied for the gridding procedure and the spectrum map is obtained by direct inverse Fourier transform (refer to (10)). By applying the image synthesis technique, spectrum map is constructed and it is shown in Fig. 3(a) and Fig. 3(b) and the positions of active sources on both are the same as those in Table 1, which means that the proposed approach locates accurate positions of active sources on the spectrum map where spectrum map in Fig. 3(a) and Fig. 3(b) are constructed by arrays in Fig. 4(a) and Fig. 4(a), respectively.

If tapering for spectrum map is considered, for instance, the 1st discrete prolate spheroidal sequence (DPSS) [16] (time-bandwidth product equals 4) is utilized, the power intensity of User 2 and User 8 are blended in Fig. 3(b) resulting from mainlobe spreading from tapering. However, this problem can be solved by applying longer baselines from Fig. 4(b) and here Fig. 5 shows the case. The power intensity of user 2 and user 8 now is recognizable because longer baselines increase resolution in spectrum map, which means that the knowledge radio resource in spectrum map becomes more accurate.

With the help of SAR technology, the proposed scheme is numerically evaluated and results show that the spectrum map constructed by the proposed solution can really provide accurate location information of active sources and radio resource intensity. In addition, only few sensors are required becomes to the other advantage. Therefore, spectrum map can compensate parts of shortcomings in the spectrum sensing technology by providing further position information of active sources when particularly links are required in cognitive radio networks.

5. CONCLUSIONS

In this paper, authors proposed a solution for construction of spectrum map which can provide knowledge of radio resource and accurate corresponding position information in wireless communications. By resorting to technology of SAR, spectrum map can be constructed by using only few sensors and thus solves the shortage of general approach by numerous sensors. Most important of all, spectrum map construction can generalize availability of CR link(s) to fully exploit radio spectrum utilization for CRN so that CRN functions such as routing and power control can be successfully developed and deployed.

6. REFERENCES

resolution techniques for radio astronomical imaging. 


