Orthogonal Frequency Division Multiplexing (OFDM) and Orthogonal Frequency Division Multiple Access (OFDMA)

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Origin of OFDM

- Parallel orthogonal transmission similar to FDM
- If realized by DFT (thus FFT), only one transmitter/receiver unit is needed
Spectrum of OFDM Signal
Historical Milestones of OFDM

- Robert Chang invented OFDM
- Steve Weinstein and Paul Ebert introduced DFT to OFDM
- Cyclic prefix was introduced against multipath fading by A. Ruiz
- Len Cimini proved OFDM working well in fading channels
- Multi-carrier technology was adopted by ADSL due to J. Cioffi
- IEEE 802.11a adopted OFDM in 1999, later ETSI HIPERLAN/2, and more after 2000 such as IEEE 802.11g.
  - Due to its high spectral efficiency and less complexity of equalizer for each sub-carrier
An Early Version of OFDM

Figures are from A. Bahai, B. Saltzberg, Multi-carrier Digital Communications, Kluwer: 1999.
Basic OFDM system
Practical Development of OFDM

- OFDM can not be practically used due to ISI/ICI caused by multipath fading destroying orthogonal property.

- In 1980’s, cyclic prefix (acting as a guard-time) was proposed to overcome fading delay spread and proven so.

![Guard Time Diagram]
OFDM Transceiver Block Diagram

Frequency selective fading is a big challenge!
Multi-carrier Communications

In early satellite communications, communications based on different carrier frequencies may go through one transponder in a communication satellite. The earth station may also combine signals with different carrier frequencies through one power amplifier to satellite, or through one low noise amplifier (LNA) from satellite. This can be considered as a sort of multi-carrier communications. When a new communication challenge to provide highly bandwidth efficient transmission for asymmetric digital subscriber line (ADSL) arises, J. Cioffi etc. developed multi-carrier technology by dividing available spectrum over the wire into many frequency sub-bands and by adjusting modulation constellations over these sub-bands based on channel situation. State-of-the-art wireless communications adopt a similar concept as multi-carrier technology known as orthogonal frequency division multiplexing (OFDM). As indicated in Chapter 1, OFDM and its multiuser version OFDMA have been widely used in the IEEE 802.11 g/a wireless LANs (also known as WiFi) and IEEE 802.16e (also known as mobile WiMAX), 3G LTE, etc. state-of-the-art wireless broadband communications.
Definition 9.1: The $N$-point DFT of a finite length sequence $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$  \hspace{1cm} (9.1)

The inverse transform called inverse DFT (IDFT) provides a way to recover the finite length sequence as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}$$  \hspace{1cm} (9.2)
FFT
OFDM with Cyclic Extension

To alleviate inter-channel interference (ICI) and inter-symbol interference (ISI)
Basic OFDM

(a) Transmitter

(b) Receiver
The original OFDM concept can be realized as follows. The transmitted signal spectrum is chosen that inter-channel interference (ICI) does not occur. More precisely, the spectrum of each sub-carrier (sub-channel) is zero at other sub-carrier frequencies. We modulate $N$ data in parallel, spaced by $\Delta t = 1/f_s$ ($f_s$ is the symbol rate) with $N$ sub-carrier frequencies. The signaling interval $T$ increases to $N\Delta t$, which suggests the system more vulnerable to delay spread from fading and channel impairments. As Figure 9.4, the transmitted waveform is

$$D(t) = \sum_{n=0}^{N-1} \{a(n) \cos \omega_n t + b(n) \sin \omega_n t\}$$

(9.3)

where $\omega_n = 2\pi f_n$, $f_n = f_0 + n\Delta f$, and $\Delta f = \frac{1}{N\Delta t}$. 
To explore the spectral efficiency of OFDM, we suppose that the symbol rate of serial data is $1/\Delta t$. The bit rate for a corresponding $M$-ary system is $\log_2 \frac{M}{\Delta t}$.

Each sub-channel transmits at a much lower rate $\log_2 \frac{M}{N\Delta t}$, due to $N$ parallel sub-channels. The total bandwidth of the OFDM system is

$$B = f_{N-1} - f_0 + 2\delta$$

(9.4)

where $f_n$ is the frequency of the $n$th sub-carrier, and $\delta$ is the one-side bandwidth of the sub-channel. The sub-carriers are orthogonally packed and uniformly spaced so that $f_{N-1} - f_0 = (N-1)\Delta f$. Since $\Delta f = \frac{1}{N\Delta t}$ and $\delta = \frac{\Delta f}{2} = \frac{1}{2N\Delta t}$, $f_{N-1} - f_0 = (1 - \frac{1}{N}) \frac{1}{\Delta t}$. As we practically reserve some extra guard for filtering by a factor of $\alpha$ (i.e. $\delta = (1 + \alpha) \frac{1}{2N\Delta t}$), the spectral efficiency $\eta$ is

$$\eta = \frac{\log_2 M}{\left(1 - \frac{1}{N}\right) + 2\delta \Delta t} = \frac{\log_2 M}{1 + \frac{\alpha}{N}} \leq \log_2 M$$

(9.5)
OFDM via FFT
From (9.3), such implementation suggests

\[ D(t) = \text{Re}\left[ \sum_{n=0}^{N-1} d(n)e^{-j\omega_n t} \right] \quad (9.6) \]

Channel fading and possible co-channel interference may distort the signal to impair signal orthogonality among sub-carriers. The fading can be modeled as

\[ Z(m) = A(m)e^{j\theta(m)} \quad (9.7) \]

where \( A(m) \) are contributed from fading statistics (such as Rayleigh) and \( \theta(m) \) are from a uniformly distributed random phase. The output data sequence is

\[
\hat{d}(i) = \frac{1}{N} \sum_{m=0}^{N-1} Z(m)D(m)e^{j\frac{2\pi}{N}bm} \\
= \sum_{m=0}^{N-1} d(n) \left[ \frac{1}{N} \sum_{m=0}^{N-1} Z(m)e^{j\frac{2\pi}{N}m(l-n)} \right] \\
= \sum_{n=0}^{N-1} d(n)z(l-n) \quad (9.8)
\]
where $z(l)$ is the inverse DFT of $Z(m)$. (9.8) implies a complex average of samples of the complex fading envelope. Without any fading, $Z(m) = 1$, and $z(l-n) = δ(l-n)$, thus $\hat{d}(l) = d(l)$. In the presence of fading,

$$\hat{d}(l) = d(l)z(0) + \sum_{n=0, n \neq l}^{N-1} d(n)z(l-n)$$

(9.9)

The second term means the inter-channel interference (ICI) and inter-symbol interference (ISI) resulted from loss of orthogonality. As a matter of fact, ICI can create great harm to OFDM communications. To ensure good communication, pilot signals are usually inserted into transmission to serve as the reference of amplitude and carrier phase, to aid the performance of receiver.
Block Diagram of OFDM
Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing

LEONARD J. CIMINI, JR., MEMBER, IEEE

Abstract—This paper discusses the analysis and simulation of a technique for combating the effects of multipath propagation and cochannel interference on a narrow-band digital mobile channel. This system uses the discrete Fourier transform to orthogonally frequency multiplex many narrow subchannels, each signaling at a very low rate, into one high-rate channel. When this technique is used with pilot-based correction, the effects of flat Rayleigh fading can be reduced significantly. An improvement in signal-to-interference ratio of 6 dB can be obtained over the bursty Rayleigh channel. In addition, with each subchannel signaling at a low rate, this technique can provide added protection against delay spread. To enhance the behavior of the technique in a heavily frequency-selective environment, interpolated pilots are used. A frequency offset reference scheme is employed for the pilots to improve protection against cochannel interference.

available spectrum, it is desirable to look for channel designs which provide good performance for both speech and data transmission, and which are also bandwidth efficient. The channel designs presented in this paper could accommodate speech or data transmission. For the narrow channel assumed, a low-bit-rate speech coder would be required. For example, a 7.5 kHz channel using the system proposed in this paper can support 8.6 kbits/s. In what follows, the channel will be assumed to be transmitting data symbols.

In a conventional serial data system, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth. Due to the bursty nature of the Rayleigh channel, several adjacent symbols may be completely destroyed during a fade. To illu-
Practical OFDM Transceiver Design
IEEE 802.11a OFDM Subcarriers

- 20M Hz bandwidth
- 52 sub-carriers (48 data subcarriers and 4 pilot subcarriers)

- 4 pilot subcarriers are located at +7, -7, +21, -21 subcarriers for channel estimation, which is critical in frequency selective fading
IEEE 802.11a Frequency Plan

- **Lower and Middle U-NII Band (5.15-5.35GHz)**
  - 8 carriers in 200M Hz
  - $\delta=30$M Hz, $\Delta f=20$M Hz
  - Lower U-NII band, max. transmission power is 40 mW (2.5mW/MHz)
  - Middle U-NII band, max. transmission power is 200 mW (12.5mW/MHz)

- **Upper U-NII Band (5.725-5.85GHz ISM, 5.75-5.85G Hz)**
  - 4 carriers in 100M Hz
  - $\delta=20$M Hz, $\Delta f=20$M Hz
  - Max. transmission power is 800 mW (50mW/MHz)
IEEE 802.11a/g Data Rates

<table>
<thead>
<tr>
<th>Data Rate (Mbps)</th>
<th>Modulation</th>
<th>Coding Rate (R)</th>
<th>Coded Bits per Sub-carrier ($N_{bps}$)</th>
<th>Coded Bits per OFDM Symbol ($N_{cbps}$)</th>
<th>Data Bits per OFDM Symbol ($N_{dbps}$)</th>
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</thead>
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<tr>
<td>6</td>
<td>BPSK</td>
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<td>24</td>
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<tr>
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<td>96</td>
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<td>4</td>
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<td>4</td>
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<td>$\frac{3}{4}$</td>
<td>6</td>
<td>288</td>
<td>216</td>
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</tbody>
</table>
Tx RF Subsystem

I/Q Modulator → Baseband Converter → Spectral Filter → PA
PA Transfer Function

Problems:
PAPR
ICI

Saturation
Desired Operation
Noise Floor

Output Power

Input Power
Baseband Design Due to RF

- Amplifier classification and distortion
  - Pre-distortion
  - Clipping
  - Adaptive predistortion
  - Coding
  - Partial transmit sequence
- Calibration
- IQ imbalance
- Phase noise
Synchronization

- **Timing**
  - Packet detection/synchronization
    - Received signal energy
    - Sliding window
    - Using preamble
  - Symbol synchronization
  - Tracking sampling clock frequency

- **Frequency**
  - Pre-DFT
  - Post-DFT

- **Carrier phase tracking**
  - Phase tracking
Channel Estimation

- Estimating frequency response of radio channel
  - Frequency domain
    - Using pilot sub-carriers and interpolation
  - Time domain
- Equalization
  - Frequency domain approach
  - Time domain approach
- Clear Channel Assessment
  - Signal quality
OFDMA

We can assign different radio blocks to different users, but to optimize sub-carriers, bits, power, remains major challenge.
Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation

Cheong Yui Wong, Roger S. Cheng, Member, IEEE, Khaled Ben Letaief, Senior Member, IEEE, and Ross D. Murch, Senior Member, IEEE

Abstract—Multiuser orthogonal frequency division multiplexing (OFDM) with adaptive multiuser subcarrier allocation and adaptive modulation is considered. Assuming knowledge of the instantaneous channel gains for all users, we propose a multiuser OFDM subcarrier, bit, and power allocation algorithm to minimize the total transmit power. This is done by assigning each user a set of subcarriers and by determining the number of bits and the transmit power level for each subcarrier. We obtain the performance of our proposed algorithm in a multiuser frequency selective fading environment for various time delay spread values and various numbers of users. The results show that our proposed algorithm outperforms multiuser OFDM systems with static time-division multiple access (TDMA) or frequency-division multiple access (FDMA) techniques which employ fixed and predetermined time-slot or subcarrier allocation schemes. We have also quantified the improvement in terms of the overall required transmit power, the bit-error rate (BER), or the area of coverage for a given outage probability.

In particular, subcarriers with large channel gains employ higher order modulation to carry more bits/OFDM symbol, while subcarriers in deep fade carry one or even zero bits/symbol. Integrated design of forward error correcting code and adaptive modulation has also been studied using BCH code and trellis coded modulation (TCM) in [8] and [9], respectively. Although both references considered only time-varying flat fading channels, the same coded adaptive modulation design can be easily applied to OFDM systems. As different subcarriers experience different fades and transmit different numbers of bits, the transmit power levels must be changed accordingly. The problem of optimal power allocation has also been studied in [10].

In this paper, we consider extending OFDM with adaptive modulation to multiuser frequency selective fading environ-
Advanced Communication Techniques

- Coded modulation
  ✓ Combining modulation and coding together
  ✓ TCM first approached theoretical limit
  ✓ Reduced state equalization

- Turbo codes

- Space-time codes and processing
  ✓ OFDM has advantages from this aspect

- Low density parity check codes

- Adaptive modulation and systems

- Signal processing to support above signaling
Phase Noise Estimation in OFDMA Uplink Communications

Chung-Kei Yu, Yi-Ching Liao, I-Hsueh Lin and Kwang-Cheng Chen

5.1 Introduction

OFDM transmission technique has been adopted in several wireless communication standards for its ability to combat channel multipath fading with relatively low complexity while providing high spectral efficiency in comparison to single carrier transmission. An OFDMA system divides the available subcarriers into groups, called subchannels, and assigns one or multiple subchannels to multiple users for simultaneous transmission. Signals from different users overlap in frequency domain but occupy different subcarriers, and the orthogonality among subcarriers prevents multiple access interference (MAI) among users.

On the other hand, OFDM is much more sensitive to carrier frequency offset and phase noise than single carrier systems [1] because of loss of orthogonality among OFDM subcarriers result in the appearance of common phase error (CPE) and inter-carrier interference (ICI). OFDMA inherits from OFDM the fact that is more sensitive to both these problems than single carrier multiple access systems. It is shown that if all users’ signals are received with equal power and all transceivers have the same phase noise spectrum, the degradation for OFDMA uplink is the same as for OFDM [2]. However, the users have distinct phase noise spectrum due to different oscillators and the base station may suffer from the near-far effect when the received signal power levels among subcarriers from different users are different. Users with higher power can consequently interfere severely with those users with lower power. On the other hand, poor phase noise spectrum (usually due to unsatisfactory oscillators) can ruin the overall system performance [19]. Therefore, phase noise is more detrimental to uplink OFDMA systems if not carefully compensated.

Various methods to suppress phase noise in OFDM systems have been proposed in the literature [3]–[5]. However, they are specifically suitable for dealing with single phase noise. To mitigate multiple phase noise in OFDMA uplink, the adopted subcarrier assignment scheme