Radio Resource Allocation for Mobile MIMO-OFDMA

Feng Seng Chu
Institute of Communication Engineering,
National Taiwan University.
Taipei, Taiwan.
B8901009@ee.ntu.edu.tw

Kwang Cheng Chen
Institute of Communication Engineering,
National Taiwan University.
Taipei, Taiwan.
chenkc@cc.ee.ntu.edu.tw

Abstract—Radio resource allocation has been widely considered in recent years to further improve communication system performance, while MIMO-OFDMA (Multiple Input Multiple Output – Orthogonal Frequency Division Multiple Access) is one of key technologies in future wireless systems. In this paper we propose an algorithm to adaptively allocate power, bandwidth and antennas among mobile users in MIMO-OFDMA system; the channel equalization was integrated into our signal model to cancel the inter-antenna and inter-carrier interference. By the QR decomposition, we can eliminate the interference and simplified the mobile channel condition to nomadic case. Our numerical results demonstrate that, the proposed algorithm can achieve near-optimal performance while maintain linear complexity simultaneously. Furthermore, the proposed algorithm still can improve system capacity even when we include practical channel estimation and prediction bias into consideration.

Keywords: MIMO, OFDMA, Equalization, Radio Resource Allocation, Mobile.

I. INTRODUCTION

According to scenarios defined by the ITU-R, the next generation wireless communication should support gigabit or higher data rate for nomadic users and hundreds of megabit data rate for users with high mobility, to meet tremendous demand of applications. Orthogonal Frequency Division Multiple Access (OFDMA) is widely considered as the multiple access technology for state-of-the-art and future wireless communication due to high-spectral efficiency and more flexibility in radio resource allocation. Multiple Input Multiple Output (MIMO) technology significantly improves the physical capacity well fits OFDM(A) while the MIMO-OFDMA is one of key technologies for broadband mobile communication systems.

It has been shown [1-2] [8] that the OFDMA system capacity can be increased by properly allocating sub-carriers among users and adjusting power on each sub-carrier according to user channel conditions. Such concept was then further extended to MIMO-OFDMA [3-7] by considering multiple-antenna interface. In some researches, antenna correlation was fixed by Alamouti code [3] or SVD (Singular Value Decomposition) [4-5] to simplify the CINR (Carrier to Interference and Noise Ratio) to SNR (Signal to Noise Ratio).

There are still other papers considering antenna selection [6] and keep the interference inside the problem formulation [7].

However, mobile channel conditions are crucial for wireless communication while the earlier researches only considered static users. On the other hand, the equivalent channel gain experienced by each subcarrier varies with equalization (pre-coding or post-coding) scheme at receiver. The common approach SVD is not appropriate for MIMO-OFDMA because we cannot use the same pre-code matrix for all users. So the singular values of channel matrix cannot be seem as the equivalent channel gain experienced by each subcarrier. We need another equalization technology when considering resource allocation of MIMO-OFDMA.

In this paper we first use the QR decomposition as our equalization scheme to find the equivalent SNR of each subcarrier, and then proposed a cross-layer design resource allocation for mobile MIMO-OFDMA. Finally, we numerically demonstrate the performance of the proposed algorithm.

II. SIGNAL MODEL

Throughout this paper, we assume the number of Tx antenna and Rx antenna are N, the number of system users is U. One Tx antenna have an OFDMA frame including T OFDM symbol, each symbol have K subcarriers.

The time domain transmitted signal vector (neglecting cyclic prefix) can be represented as (1), where \( \alpha \), \( r \) and \( k \) are Tx antenna, symbol and subcarrier index. \( F \) is FFT matrix.

\[
\mathbf{x} = \mathbf{\Psi}^H \mathbf{F}^H \begin{bmatrix}
0 & 0 & 0 \\
0 & F^H & 0 \\
\vdots & \vdots & \ddots \\
0 & 0 & \cdots F^H
\end{bmatrix} \mathbf{x}
\]

(1)

After multiplying channel matrix \( \mathbf{D}_u \) belonging to user \( u \), the received signal vector of user \( u \) can be represented as (2), where \( u \) is user index and \( \mathbf{w}_u \) is time domain additive noise vector.
\[\mathbf{r}_u = \mathbf{D}_u \Psi^H \mathbf{X} + \mathbf{w}_u \]  

(2)

By IFFT, the demodulated signal vector can be represented as (3), where \( \Omega_u \) and \( \mathbf{W}_u \) are equivalent channel matrix and frequency domain additive noise vector.

\[ \mathbf{Y}_u = \Psi \mathbf{D}_u \Psi^H \mathbf{X} + \Psi \mathbf{W}_u = \Omega_u \mathbf{X} + \mathbf{W}_u \]  

(3)

Due to the antenna correlation and time-varying channel, non-diagonal terms of the equivalent channel matrix \( \Omega_u \) are not all zero; this phenomenon is the so-called inter-antenna interference (IAI) and inter-carrier interference (ICI).

Since for any matrix \( \Omega_u \), there must be one unitary matrix \( \mathbf{Q}_u \) and one upper triangular matrix \( \mathbf{R}_u \) such that \( \Omega_u = \mathbf{Q}_u \mathbf{R}_u \). We can multiply the demodulated signal \( \mathbf{Y}_u \) by \( \mathbf{Q}_u^H \) as (4).

\[ \mathbf{Y}^*_u = \mathbf{R}_u \mathbf{X} + \mathbf{Q}_u^H \mathbf{W}_u = \mathbf{R}_u \mathbf{X} + \mathbf{W}^*_u \]  

(4)

Since \( \mathbf{R}_u \) is an upper triangular matrix, we can find each component of \( \mathbf{Y}^*_u \) sequentially start from the last element of \( \mathbf{X} \), and each diagonal terms of \( \mathbf{R}_u \) was used as the equivalent SNR of each data carrier.

III. PROBLEM FORMULATION

Since we have cancelled IAI and ICI by the QR decomposition, the SNR of subcarrier \( k \) in symbol \( t \) on Tx antenna \( \alpha \) for user \( u \) can be represented as (5), where \( r_{u,a,t,k} \) are diagonal terms of upper triangular matrix \( \mathbf{R}_u \) and \( p_{a,t,k} \) are the power allocated to subcarrier \( k \) in symbol \( t \) on antenna \( \alpha \). \( B \) is total signal bandwidth and \( N_0 \) is the PSD of additive noise.

\[ \text{SNR}_{u,a,t,k} = \left[ \frac{r_{a,t,k}^2 p_{a,t,k}}{B/k N_0} \right]^2 \]  

(5)

By (5), the radio resource allocation can be mathematically formulated as (6), where \( 0 \leq \omega_{u,a,t,k} \leq 1 \) are the variables representing subcarrier allocation.

\[ T = \max_{p_{a,t,k}, \omega_{u,a,t,k}} \sum_{u=0}^{U-1} \sum_{a=0}^{N-1} \sum_{t=0}^{T-1} \sum_{k=0}^{K-1} \omega_{a,t,k} \log_2 \left( 1 + \text{SNR}_{a,t,k} p_{a,t,k} \right) \]  

(6)

Subject to

(i) \[ \sum_{a=0}^{N-1} \sum_{t=0}^{T-1} p_{a,t,k} \leq P_T \]  

(TotalPower Constraint)

(ii) \[ p_{a,t,k} \geq 0 \text{ for all } a, t, k \]  

(Non-negative Constraint)

(iii) \[ \sum_{a=0}^{N-1} \omega_{a,t,k} \leq 1 \text{ for all } a, t, k \]  

(FractionConstraint)

(iv) \[ \frac{R_u}{T} = f(\gamma_u) = \frac{1}{\sum_{a=1}^{U-1} f(\pi_{a,t}, \tau_a)} \]  

(FairnessConstraint)

where \( R_u \) is the capacity allocated to user \( u \), which is formulated as (6).

\[ R_u = \sum_{a=0}^{N-1} \sum_{t=0}^{T-1} \sum_{k=0}^{K-1} \omega_{a,t,k} \log_2 \left( 1 + \text{SNR}_{a,t,k} p_{a,t,k} \right) \]  

(7)

In the above algorithm, the object function is to maximize sum capacity. Constraint (i) is the total power constraint which limit the total power under \( P_T \). Constraint (ii) is non-negative constraint since \( p_{a,t,k} \) is power. Constraint (iii) is the fraction constraint. In our algorithm, one subcarrier can be divided into several pieces (e.g., 50%, 20% and 30%) and allocated to different users to further utilize user diversity and maintain the proportional fairness among them. Such scheme can be realized by adaptive modulation/coding. Since we allow the BS divides one sub-carrier into several pieces and allocate them to different users. We expect that, for one subcarrier, the sum of all pieces of this subcarrier should smaller than one. Constraint (iv) is the fairness constraint using proportional criterion, where \( f(\gamma) \) is fairness function that includes user priority \( \pi_u \) and channel quality \( \tau_u \) as parameters. The fairness function is actually configured by system designers.

IV. OPTIMIZATION ALGORITHM

In order to find the global optimal power and subcarrier allocation in (6), we need to run a multi-variable non-linear programming. Apparently, the complexity of such optimization is quite high. In this paper, we divide the whole optimization into subcarrier allocation and power allocation to reduce the complexity but maintain the near-optimal performance.

A. Optimal Subcarrier Allocation

We first assume that the power allocation \( p_{a,t,k} \) are known and formulate the subcarrier allocation as (7).

\[ T = \max_{\omega_{u,a,t,k}} \sum_{u=0}^{U-1} \sum_{a=0}^{N-1} \sum_{t=0}^{T-1} \sum_{k=0}^{K-1} \omega_{a,t,k} \log_2 \left( 1 + \text{SNR}_{a,t,k} p_{a,t,k} \right) \]  

Subject to

(i) \[ \sum_{a=0}^{N-1} \omega_{a,t,k} \leq 1 \text{ for all } a, t, k \]  

(FractionConstraint)

(ii) \[ \frac{R_u}{T} = \frac{f(\gamma_u)}{\sum_{a=1}^{U-1} f(\pi_{a,t}, \tau_a)} = \gamma_u \]  

(FairnessConstraint)

This problem formulation is very similar to (6) except constraint for power allocation. Since \( p_{a,t,k} \) in (7) are assumed known now, all variables in \( \log \) function are known. We can further simplify the (7) into a constraint linear programming (8).

\[ T = \max_{w} F^T W \]  

Subject to

(i) \[ \sum_{a=0}^{N-1} \omega_{a,t,k} \leq 1 \text{ for all } a, t, k \]  

(FractionConstraint)

(ii) \[ \frac{R_u}{T} = \frac{f(\gamma_u)}{\sum_{a=1}^{U-1} f(\pi_{a,t}, \tau_a)} = \gamma_u \]  

(FairnessConstraint)
where
\[ F = \log_2 \left( 1 + \text{SNR}_{u,a,i,k} \left( p_{u,a,i,k} \right) \right) \]
\[ W = \left[ \omega_{u,a,i,k} \right] \]

Such optimization can be easily solved in a linear complexity.

B. Sub-optimal Power Allocation

Since we allow users to divide one subcarrier into several pieces, the traditional water-filling optimal power allocation scheme cannot be applied directly. But by detailed observation of numerical results, we find that, one subcarrier is divided only due to fairness constraint. In each simulation, there are only \( U-1 \) subcarriers were divided for different users. Since the number of divided subcarrier is quite smaller than the total number of subcarriers. We still use the concept of water-filling for power allocation but slightly modified. For subcarriers those were divided for different users, we just let the channel gain of user who has the largest fraction of this subcarrier as the channel gain of this subcarrier and obtain the following algorithm.

\[ p_{u,a,i,k} = \left\{ \frac{P_t}{NTK \sum_{k=0}^{K-1} \sum_{i=0}^{I-1} \sum_{u=1}^{U} \frac{B/K N_0}{\omega_{u,a,i,k}} - \frac{B/K N_0}{\omega_{u,a,i,k}}} \right\}^+ \]

Due to the non-linearity of power allocation, in power allocation we relax the fairness constraint and only maximize the system sum capacity. That is why in next part we need one more step to re-maintain user fairness.

C. The Final Algorithm

By the introduced optimal subcarrier allocation and sub-optimal power allocation. We list the resulted three-step algorithm in the following.

step1: Optimal subcarrier allocation under uniform power allocation (Consider user fairness)

step2: Sub-optimal power allocation under subcarrier allocation in step1. (Without considering fairness)

step3: Optimal subcarrier allocation under power allocation in step2. (Re-maintain user fairness)

In the following section, we start to introduce the scenario we used to evaluate the proposed algorithm.

V. EVALUATION SCENARIO

In this section we introduce the methodology we used to evaluate the proposed algorithm such as practical consideration (Tx channel prediction error, Rx channel estimation error), comparison systems and channel model.

A. Practical Consideration

In practical operations, downlink channel state information was estimated by receiver and feedback to transmitter. According to the feedback information, the transmitter predicts channel condition of this receiver and allocates radio resource to this user appropriately. It is apparent that the receiver cannot estimate the channel state information perfectly, and the transmitter also cannot predict the channel condition without error. In this part we consider the theoretical impact on each subcarrier SNR when there is estimation bias at receiver.

In (3) and (4), we use \( \Omega_u \) as “correct” equivalent channel matrix. Now, we assume the “estimated” equivalent channel matrix as \( \hat{\Omega}_u \) and \( \hat{\Omega}_u = \hat{Q}_u \hat{R}_u = \Omega_u + \Delta_u = \hat{Q}_u \hat{R}_u + \Delta_u \). The demodulated signal become to (10).

\[ \hat{Q}_u^H Y_u = \hat{R}_u X - \hat{Q}_u^H \Delta_u X + \hat{Q}_u^H W_u \]

In addition to the noise term \( \hat{Q}_u^H W_u \), we find another error term \( \hat{Q}_u^H \Delta_u X \) induced by channel estimation bias at receiver. In order to evaluate the proposed algorithm in a more practical environment, we replace the SNR in (5) by CINR (Carrier to Interference and Noise Ratio) in (11) when computing system capacity, where \( P \) is power allocation vector \( P = [p_{u,a,i,k}] \).

\[ \text{SNR}_{u,a,i,k} = \frac{\left[ \frac{P_{u,a,i,k}}{\omega_{u,a,i,k}} p_{u,a,i,k} \right]^2 + \frac{B/K N_0}{\omega_{u,a,i,k}}} \]

\[ \left[ \frac{\Delta_u}{\Omega_u} \right]_{\text{element-wise}} \in \left[ \frac{-\eta_f / 2}{\eta_f / 2} \right] \]

In our simulation, we allocate resource by predicted channel matrix and compute the resulted capacity by including estimation bias.

B. Comparison Algorithms

In this paper we select two existing algorithms for comparisons. The first one is the radio resource allocation algorithm proposed by Zukang and Jeffrey [2] which includes one sub-optimal subcarrier allocation and one optimal power allocation. The problem formulation in [2] is almost the same as (6) and that is why we select it as comparison system. The second one is the OFDM-based round robin TDMA strategy (with optimal power allocation) where each user takes turn to transmit their signals.

The performance of algorithm [2] is very close to optimal, so any improvement from this algorithm is treasury. Actually, the proposed algorithm not only improves a little capacity from algorithm [2], but also reduces the computing complexity.

C. Fairness Consideration

Since both the proposed algorithm and comparison algorithm [2] uses proportional fairness criterion. We
normalize the result sum capacity by the min-max fairness index [10] as (13) to demonstrate the capacity improvement and fairness maintenance simultaneously.

\[
I_{\text{min-max}} = \max_u \left( \frac{R_u}{y_u} \right) \div \min_u \left( \frac{R_u}{y_u} \right)
\]

(13)

\[
C_{\text{fair normalized}} = \frac{C}{I_{\text{min-max}}} = \frac{\sum R_u}{\max I_{\text{min-max}} - \min I_{\text{min-max}}}
\]

In all cases of section VI, we use such fair normalized capacity as the main tool to demonstrate the performance of proposed and comparison systems.

D. Channel Model and Simulation Parameter

In this paper we adopt the spatial channel model in IEEE 802.16m evaluation methodology [9] in our simulation. We use the urban macro-cell as the evaluation scenario. The simulation parameters were listed in Table 1.

<table>
<thead>
<tr>
<th>TABLE I. SIMULATION PARAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sampling Rate</strong></td>
</tr>
<tr>
<td>User (U)</td>
</tr>
<tr>
<td>Symbol of Each Frame (T)</td>
</tr>
<tr>
<td>Antenna of Tx and Rx (N)</td>
</tr>
<tr>
<td>Sub-carrier number (K)</td>
</tr>
<tr>
<td>Cluster in CDL model</td>
</tr>
<tr>
<td>Doppler Frequency of Each User</td>
</tr>
<tr>
<td>Signal to Noise Ratio (SNR)</td>
</tr>
<tr>
<td>Prediction error (uniform) ((\eta_p))</td>
</tr>
<tr>
<td>Estimated error (uniform) ((\eta_e))</td>
</tr>
<tr>
<td>Priority of Each User ((\pi_k))</td>
</tr>
<tr>
<td>Fairness Function</td>
</tr>
</tbody>
</table>

In part A and B of section VI, we fixed the prediction and estimation error as \(\eta_p = 10\%\) and \(\eta_e = 0.1\%\) to observe the capacity improvement from comparison systems. In part D and E, we adjust \(\eta_p\) and \(\eta_e\) to observe the robustness of the proposed algorithm.

VI. NUMERICAL RESULT

In this section we demonstrate the capacity improvement, fairness maintenance and robustness of the proposed algorithm and comparison systems.

A. Capacity Among Steps of the Proposed Algorithm

In this part we observe the capacity among the three steps of the proposed algorithm. For each step, we consider three cases of antenna = 1, 2 and 4. All the results were plotted in Figure 1. From Figure 1 we can see that the performances of the three steps are very closed, especially step 1 and step 3. It is very intuitive since we divide the sum capacity by fairness index; the capacity of step 2 must be smaller than step 1 and 3 due to unfair power allocation.

Since the performance of the step 1 is better than step 2 and comparable to step 3. It seems we need only the optimal subcarrier allocation, but not power allocation anymore. In the following part, we only use the step 1 to compare to other algorithms.

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B. Capacity Improvement from Comparison System

In this part we observe the capacity improvement of the proposed algorithm from the two selected comparison systems.

From Figure 2, we can see that the fair normalized capacity of the proposed algorithm is a little better than the algorithm proposed in [2], and the more the subcarriers or antennas the larger the improvement. It has been shown [2] that the capacity of algorithm [2] is very close to optimal allocation. So it is admirable that the proposed algorithm can further improve the sum capacity without increase complexity.

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In addition to the comparisons among proposed algorithm and algorithm [2], we also can see that both the two algorithms are quite better than the round-robin algorithm.
C. Complexity Comparison

It has been shown in [2] that the complexity of the algorithm [2] is $O(UNTK)$. Since the step 1 (optimal subcarrier allocation) of the proposed algorithm is linear programming, its complexity is also $O(UNTK)$. So the complexity of the proposed algorithm and algorithm [2] are similar.

D. Effect of Estimation Bias

In this part we observe a more practical case considering channel estimation bias. In Figure 4 we can see that, as the increased of estimation bias, the behavior of those curves change from AWGN to interference-limit case. The higher the SNR, the higher the interference induced among subcarriers. It is reasonable since we cannot equalize (cancel) the interference among antennas and subcarriers due to channel estimation bias.

![Figure 3](image-url)  
**Figure 3.** Effect of Channel Estimation Bias (Antenna = 2 in all cases)

E. Effect of Prediction Bias

In this part we observe the effect of channel prediction. We select $\eta_p$ as 1%, 10% and 20% in Figure 4. It seems that all the three algorithms are quite robust to such prediction bias.

![Figure 4](image-url)  
**Figure 4.** Effect of Channel Prediction Bias (Antenna = 2 in all cases)

VII. Conclusion

In this paper we proposed a radio resource allocation algorithm for downlink mobile MIMO-OFDMA. We include the channel equalization into our study to cancel the inter antenna and inter carrier interference. In our numeric results based on IEEE 802.16m scenarios, we demonstrate that almost all performance improvement came from subcarrier allocation, and we can further improve system capacity by the proposed subcarrier allocation from the selected comparison algorithms under fairness constraint without increasing complexity. We also demonstrate that, the performance improvement do not vanish even in a more practical environment considering channel estimation and prediction bias. All of those results show that the proposed algorithm may be one choice when considering next generation wireless communication systems.

REFERENCES


