Multiple Systems Sensing for Cognitive Radio Networks over Rayleigh Fading Channel

Chung-Kai Yu
Graduate Institute of Communication Engineering
National Taiwan University
Taipei, Taiwan
r95942061@ntu.edu.tw

Kwang-Cheng Chen
Graduate Institute of Communication Engineering
National Taiwan University
Taipei, Taiwan
chenkc@cc.ee.ntu.edu.tw

Abstract—Cognitive radios (CRs) with capability of spectrum sensing to fully utilize radio spectrum have been considered as a key technology toward future wireless communications. We can further leverage CRs to form cognitive radio networks (CRNs). When we consider sensing for CRNs, it is critical to identify the potential primary and secondary systems for construction of time-varying CRNs. In this paper we propose a general sensing algorithm to sense the spectrum usage and identify active systems under a multiple-systems coexisting environment over Rayleigh fading channel. We exploit unique characteristics of systems consisting of fundamental frequency, power spectrum density and four-order cumulant to accomplish the multiple systems sensing. Our algorithm can also be applied by Detect and Avoid (DAA) for UWB with a priori knowledge of system parameters in active systems.

Keywords- Spectrum Sensing, Cognitive Radio Networks, Multiple Systems Sensing, Rayleigh Fading, Detect and Avoid.

I. INTRODUCTION

The concept of cognitive radio (CR) was first proposed by Mitola in 1999 [1] and has grasped tremendous attention because of the spectrum shortage. Spectrum sensing of cognitive radios conventionally distinguishes link level availability (i.e. idle and thus cognitive radio can use, or used and cognitive radio cannot use.) Further incorporating cognitive radios to form cognitive radio networks (CRNs), spectrum sensing should achieve networking level functions in addition to conventional spectrum sensing. In cognitive radio networks, cognitive radios can communicate with either primary systems (maybe more than one primary system) or other cognitive radios (secondary systems) to optimize entire network efficiency over radio spectrum [2]. Therefore, in addition to sensing the idle status of spectrum, identification of multiple systems is critical for establishing connections and building up the cognitive radio networks.

Spectrum sensing techniques traditionally include energy detection, CP existence, pilot detection, spatiotemporal sensing, etc [3]-[7]. In [17], sensing under a multiple coexisting environment such as 2.4GHz ISM band is considered by distributed classification. However, with inter-system interference [8][9], traditional techniques are not enough and thus a more reliable and general multiple systems sensing algorithm is needed to overcome this challenge. In this paper, we propose a methodology exploiting the system-specific characteristics to identify spectrum utilization status and to identify multiple active systems, over uncorrelated Rayleigh fading channels. In addition to energy detection and carrier locking, we identify the fundamental frequencies of candidate communication systems periodically filtered by pulse shaping filters as mentioned in [3]. To accomplish the multiple systems sensing, we have to further exploit the unique power spectrum density pattern of systems. If the additive noise is colored Gaussian with unknown covariance matrix, the power spectrum density pattern methodology might not apply. We may further make use of high-order statistics that cumulants are blind to any kind of a Gaussian process [10] to ensure the success of our multi-system sensing for cognitive radio networks.

The rest of this paper is organized as follows. We discuss the multiple systems sensing and propose the general multiple systems sensing algorithm in Section II. Numerical results for multiple coexisting systems at 2.4GHz ISM band are provided in Section III. The conclusion is given in Section IV.

II. PROBLEM FORMULATION

A. Cognitive cycle

The proposed cognitive cycle for cognitive radio networks is shown in Figure 1. Different from conventional cognitive cycle [3][6], sensing for networks/systems and optimization for network efficiency over radio spectrum are included. Since the network variation is usually much lower than spectrum usage variation, networks/systems sensing (or says networks/systems discovery) can be performed with longer sensing cycle than conventional spectrum sensing (spectrum holes discovery). In addition, rather than the work of spectrum efficiency optimization of conventional CR, network efficiency optimization is critical for cognitive radio networks. In this paper we will focus on the network/systems sensing, especially for multiple systems coexisting in spectrum.
B. System Model

Assume that there are \( Q \) candidate communication systems. Suppose that the transmitted signal of each system is going through a flat uncorrelated Rayleigh fading channel, that is, each with an independent complex amplitude \( a_i = |a_i| e^{j\delta_i} \) where amplitude \( |a_i| \) is Rayleigh distributed with \( E(|a_i|^2) = \gamma_i^2 \) and phase \( \delta_i \) is uniformly distributed over \([0,2\pi]\). In addition, a white Gaussian noise \( w(t) \) with zero mean and variance \( \sigma_w^2 \) is added to the received radio signal. Suppose the activities of systems are unchanged during the period of each sensing. With the assumption of \( P \) active systems (\( P \leq Q \)) the received radio signal can be expressed as

\[
 r(t) = \text{Re}\left\{ \sum_{i=1}^{P} a_is_i(t) + w(t) \right\} = \text{Re}\{y(t) + w(t)\} \tag{1}
\]

where \( s_i(t) \) is the signal of \( i \)-th active system and

\[
y(t) = \sum_{i=1}^{P} a_is_i(t) \tag{2}
\]

The problem of multiple systems sensing is to determine the number of active systems, \( P \), and identify them, respectively.

III. MULTIPLE SYSTEMS SENSING

The block diagram of general sensing algorithm is shown in Figure 2. We summarize our general systems sensing algorithm for multiple coexisting environments as follows:

0. Energy detection and carrier locking to initiate the algorithm.
1. Square the received radio signal and filter it by a narrowband bandpass filter containing all potential fundamental frequency. Detect the fundamental frequencies and identify the corresponding systems.
2. Estimate power spectrum density of target spectrum.
3. If the result of step 1. is none, end; otherwise, go to step 4.
4. If the covariance matrix of additive noise is known, go to step 5; otherwise, go to step 6.
5. Perform SVD of the spectrum estimation result and identify systems. End.

Figure 2. Block diagram of general sensing algorithm for multiple systems

A. Fundamental Frequency

Because of pulse shaping in digital communication systems, there are energy peaks in the reciprocal of symbol period (baud rate) and in its harmonics. We define the lowest frequency (baud rate) with energy peak as the fundamental frequency. Suppose that these \( P \) active systems are digital communication systems with fundamental frequency characteristic. The transmitted signal of \( i \)-th system can be written in the form

\[
s_i(t) = \sum_{n=-\infty}^{\infty} x_{ii}(t-nT_i) e^{j(2\pi f_{i,c}t+\alpha_i)}
\]

for \( i = 1,2,\ldots,P \) where \( \{x_{ii}\} \) is the data sequence, \( h_i(t) \) is the impulse response of the pulse-shaping filter with fundamental frequency response \( H_i(j\omega) \). \( T_i \) is the symbol duration, \( \tau_i \in [0,T_i) \) and \( \alpha_i \in [0,2\pi) \) is the time offset and the phase offset, respectively (both are regarded as constants during sensing), \( f_{i,c} \) is the carrier frequency. To be specific, we assume that \( \{x_{ii}\} \) are zero mean variance \( \sigma_i^2 \) stationary sequences with statistically independent and identically distributed elements.

There are various methods to extract the fundamental frequency information [15] and in this paper we adopt the nonlinear spectral line method with squared magnitude followed by a narrowband bandpass filter. Assuming \( y(t) \) with equal variance in real and imaginary parts and independent of \( w(t) \). It is easy to show that

\[
E[r^2(t)] = 1/2 \cdot E[|y(t)|^2 + |w(t)|^2]
\]

Thus, we can obtain (Appendix I)

\[
E[r^2(t)] = \frac{1}{2} \sum_{i=1}^{P} \frac{\sigma_i^2 \gamma_i^2}{T_i} Z_{i,0} + \frac{1}{2} \sigma_w^2 \tag{4}
\]

where

\[
Z_{i,m} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_i(j\omega) \cdot H_i^\prime\left(-j\left(\frac{2mn}{T_i} - \theta\right)\right) d\theta \tag{5}
\]

Therefore the squared signal \( r^2(t) \) can be decomposed as

\[
r^2(t) = E[r^2(t)] + \varepsilon(t)
\]

\[
= \frac{1}{2} \sum_{i=1}^{P} \frac{\sigma_i^2 \gamma_i^2}{T_i} Z_{i,0} + \frac{1}{2} \sigma_w^2 + \varepsilon(t) \tag{6}
\]

where \( \varepsilon(t) \) is the disturbance term with zero mean.

We can observe spectral lines at frequency \( 1/T_i \) in (6). After filtering the signal with a narrowband bandpass filter containing all potential fundamental frequencies, we can detect these tones to identify the corresponding systems.
B. Power Spectrum Density Pattern

Assume the power spectrum density of each candidate system in the target spectrum is known and portioned into \( M \) equal-bandwidth sub-bands. When only the power spectrum density shape is concerned, we can model the transmitted signal in discrete-time as

\[
s_i[n] = u_i[n] s_i[n] \quad i = 1, 2, \ldots, P
\]

where \( u_i[n] \) is from white noise with variance \( \sigma_i^2 \) and \( s_i[n] \) has a frequency response \( H_i(e^{j\omega}) \) that \( [H_i(e^{j2\pi k/M})]^2 = P_i(e^{j2\pi k/M})^2 \) are real and known. Thus, the received signal in discrete-time can be written as

\[
r[n] = \sum_{i=1}^{P} a_is_i[n] + w[n] = \sum_{i=1}^{P} a_i u_i[n] s_i[n] + w[n]
\]

Let

\[
p_i = [P_i(1) \: P_i(e^{j2\pi/M}) \: \ldots \: P_i(e^{j2\pi(M-1)/M})]^T
\]

be the power spectrum density pattern of \( i \)-th candidate system and assume that these power spectrum density patterns \( \{p_i\}_{i=1}^{P} \) are linear independent. Many well-known spectrum estimation methods such as periodogram, Blackman-Tukey method, Bartlett-Welch method, multitaper, etc, are suitable for different spectrum estimation requirements [16]. Assume a large observation length is available and thus the power spectrum estimation error can be neglected. The power spectrum estimation result can be expressed in vector form

\[
\hat{p} = \sum_{i=1}^{P} \gamma_i^2 \sigma_i^2 p_i + w = S \cdot h + w
\]

where

\[
\hat{p} = [\hat{P}(1) \: \hat{P}(e^{j2\pi/M}) \: \ldots \: \hat{P}(e^{j2\pi(M-1)/M})]^T
\]

and \( \hat{P}(e^{j2\pi k/M}) \) is the estimation at frequency \( \omega_k = 2\pi k/M \),

\[
S = [p_1 \: p_2 \: \cdots \: p_P]_{MxQ}
\]

is the power spectrum pattern matrix,

\[
w = [W(1) \: W(1) \: \cdots \: W(M-1)]^T
\]

is the noise contribution to power spectrum (assume that it can be estimated accurately), and

\[
h = [\gamma_1^2 \sigma_1^2 \: \gamma_2^2 \sigma_2^2 \: \cdots \: \gamma_P^2 \sigma_P^2]^T
\]

(16)

is the received power vector.

Since \( \{p_i\}_{i=1}^{P} \) are linear independent, \( S \) is a \( M \times Q \) matrix of rank \( Q \). By performing singular value decomposition (SVD), \( S \) becomes

\[
S = U \Lambda V^T
\]

(17)

where \( U \) and \( V \) are an \( M \times M \) orthogonal matrix and an \( Q \times Q \) orthogonal matrix, respectively, and \( \Lambda \) is a \( M \times Q \) matrix with \( (i,j) \)-entry

\[
\begin{cases}
\lambda_i & \text{for } i = 1, 2, \ldots, Q \\
0 & \text{otherwise}
\end{cases}
\]

(18)

where \( \{\lambda_i\}_{i=1}^{Q} \) are the singular values of \( S \). Therefore we can solve \( h \) as follows,

\[
VA^T (\hat{p} - w) = VA^T UAV^T h = h
\]

(19)

where \( A \) is a \( Q \times M \) matrix with \((i,j)\)-entry

\[
\begin{cases}
\frac{1}{\lambda_i} & \text{for } i = 1, 2, \ldots, Q \\
0 & \text{otherwise}
\end{cases}
\]

(20)

and thus \( A^TA = I_Q \).

Ideally \( h \) contains nonzero elements only when the corresponding systems are active. In practice, \( h \) is always nonzero because of estimation errors. A heuristically serial search can be applied here. Arrange the elements of \( h \) in decreasing order, \( h_i \geq h_j \geq \cdots \geq h_Q \). Compute the ratio

\[
\frac{\sum_{i=1}^{h} k_i}{\sum_{i=1}^{Q} k_i}
\]

(21)

from \( P = 1 \) and stops when the ratio exceeds a predetermined threshold. When the search stops, the number of active systems is determined as \( \hat{P} \) and the corresponding elements and systems are identified.

C. Fourth-Order Cumulant

The discussion in Section III-B is based on the additive Gaussian noise with known covariance matrix. However, if the additive noise is colored Gaussian with unknown covariance matrix, the power spectrum density pattern methodology might not be applied. In this case, high-order statistics that cumulants are blind to any kind of a Gaussian process [10] becomes an useful characteristic to ensure the success of our multi-system sensing as provided in [13]. Suppose the transmitted signal \( s_i[n] \) is modeled as the form of (9) where \( u_i[n] \) becomes a stationary, white, non-Gaussian random process with fourth-order cumulant \( \rho_i \) and \( s_i[n] \) is with the same assumption that \( p_i \) is known and \( \{p_i\}_{i=1}^{Q} \) are linear independent. Define the elements in the \( M \times M \) trispectrum matrix \( C \) as

\[
c_{ij} = T(\omega_i, -\omega_i, \omega_j) \quad 1 \leq i, j \leq M \quad \omega_k = 2\pi (i-1) / M
\]

(22)

where

\[
T(\omega_1, -\omega_2, \omega_3) = \sum_{i=1}^{P} \gamma_i^4 \rho_i H_i(\omega_1) H_i(\omega_2) H_i(\omega_3) H_i(-\omega_1 - \omega_2 - \omega_3)
\]

(23)

is the received trispectrum. Therefore we can express \( C \) as

\[
C = \sum_{i=1}^{P} \gamma_i^4 \rho_i p_i p_i^T = \text{STST}
\]

(24)

where \( S \) is the source spectrum matrix defined as

\[
S = [p_1 \: p_2 \: \cdots \: p_Q]_{MxQ}
\]

(25)

and \( \Gamma \) is a \( Q \times Q \) diagonal matrix with only \( P \) nonzero diagonal elements. Ideally \( C \) is a real symmetric matrix with rank \( P \) and the number of active systems can be determined by computing the rank of \( C \). Fortunately, in practice the estimation of \( C \) is always full rank because of estimation error. By performing an eigendecomposition on the estimation of the
The trispectrum matrix, \( \hat{C} \), can be written as

\[
\hat{C} = \sum_{m=1}^{M} \lambda_m g_m g_m^H = G^2 G^H
\]

with the eigenvalues arranged in decreasing order,

\[
\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M
\]

The signal subspace is spanned by the vectors \( \{q_i\}_{i=1}^{M} \) and the noise subspace is spanned by the vectors \( \{q_i\}_{i=M+1}^{Q} \). Therefore the number of active systems and system identification can be proceeded by the serial search and MUSIC algorithm, respectively. We can compute the MUSIC pseudospectrum

\[
\hat{R}_{music}(\nu) = \frac{|p^H_p|^2}{\sum_{m=Q+1}^{M} |p^H_q_m|^2}
\]

and the systems with the corresponding \( P \) largest values are selected and are identified as active ones.

Ideally the trispectrum matrix in (23) is not affected by the power of additive Gaussian noise \( \sigma_w^2 \), which means that the fourth-order cumulant methodology is expected to perform well in low SNR environment. On the other hand, to estimate the trispectrum matrix \( C \) accurately, a large convergence time is required, which means that we need longer sensing time.

### Table I. System Parameters

<table>
<thead>
<tr>
<th>System</th>
<th>Carrier Frequency</th>
<th>Fundamental Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>802.11b</td>
<td>2412 MHz, 2437 MHz, 2462 MHz</td>
<td>11 MHz</td>
</tr>
<tr>
<td>802.11g</td>
<td>2412 MHz, 2437 MHz, 2462 MHz</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Bluetooth</td>
<td>Not Fixed</td>
<td>1 MHz</td>
</tr>
<tr>
<td>Microwave Oven</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

### IV. NUMERICAL RESULTS

In this paper we consider the environment with multiple coexisting systems at 2.4GHz ISM band, which includes WLAN (802.11b and 11g), Bluetooth and the microwave oven as potential active systems [14] in our simulation. The flow chart of multiple systems sensing for this case is shown in Figure 3.

The system parameters are listed in Table 1. When the energy detection indicates there are active systems in spectrum, we try to lock the carrier frequency 2412, 2437, 2462 MHz. If some carrier frequencies are locked, we say that 802.11b or/and 802.11g systems exist in the corresponding channels. If none is locked, we say there is no active 802.11b or 802.11g system. Next we apply the fundamental frequency methodology. Since squaring a broadband RF spectrum in analog fashion needs very efficient nonlinear devices, which might be not realistic, we can divide the frequency band into several parts to proceed the sensing. The fundamental frequency methodology can help us to determine the existence of 802.11b, 802.11g and Bluetooth systems since they are with different fundamental frequency. Figure 4 shows spectrum magnitude after squarer with 802.11b, 802.11g and Bluetooth coexisting under SNR=10 dB (assuming these three systems with equal signal power). Three tones at 1, 11 and 20 MHz are observed, which are corresponding to the fundamental frequencies of these systems. Note that because the spectrum magnitude is mainly determined by \( Z_{r,1} \) under the same signal power condition the system with a higher fundamental frequency will have a lower spectrum magnitude after squarer (assuming with the same roll-off factor). On the other hand we can observe that the disturbance is less in higher spectrum.

After spectrum estimation, we can obtain the spectrum usage status. If we know the power spectrum density of microwave oven, the power spectrum density pattern methodology can be used. Figure 5 shows the estimation of power spectrum density by Welch’s method. Here we assume that the power spectrum density pattern of the microwave oven is a rectangular shape from 2415 MHz to 2465 MHz and the 802.11g is active with central frequency 2437 MHz. We can see that when the spectrums of these systems are overlapped, the estimation results are summmed by the power spectrum density patterns of these systems and thus we can perform the SVD to detect the activity of systems.
On the other hand, if the power spectrum density of microwave oven is unknown, assuming the microwave oven interference as an additive Gaussian noise the fourth-order cumulant methodology can be applied to identify other systems with non-Gaussian signals.

V. CONCLUSION

In this paper, we consider a more general cognitive radio networks scenario and provide a general reliable sensing algorithm that can obtain spectrum usage status and system identification in a multiple systems/networks coexisting environment in harsh fading channels, which is beyond the conventional spectrum sensing purpose of cognitive radios.

With different a priori information, we can reliably achieve multiple system identification. This algorithm can also be applied to Detect and Avoid (DAA) for Ultrawideband communications. Since our multiple systems sensing algorithm can help to identify active systems, such a priori knowledge such as spectrum usage of these systems, DAA becomes convenient to operate.

APPENDIX I

Substituting (6) in (3), we can obtain

\[ r(t) = \text{Re} \left\{ \sum_{i=1}^{p} \sum_{n=-\infty}^{\infty} a_i \sum_{m=-\infty}^{\infty} x_{i,n} h_i(t-nT_i - \tau_i) e^{j(2\pi f_i t + \alpha_i)} + w(t) \right\} = \text{Re} \{ y(t) + w(t) \} \]

(29)

Since \( E[r^2(t)] = 1/2 \cdot E[|y(t)|^2 + |w(t)|^2] \) and

\[ E[|y(t)|^2] = E\{ (y(t) - y^*(t))^2 \} \]

\[ = \sum_{i=1}^{p} \sum_{n=\infty}^{\infty} E\{a_i a_i^* \} \sum_{m=\infty}^{\infty} E\{x_i x_i^* \} \]

\[ \times h_i(t-nT_i - \tau_i) h_i^*(t-nT_i - \tau_i) \]

\[ \times e^{j(2\pi f_i t + \alpha_i - 2\pi f_i t + \alpha_i)} \]

\[ = \sum_{i=1}^{p} \sigma_i^2 \gamma_i^2 \sum_{n=-\infty}^{\infty} h_i(t-nT_i - \tau_i) h_i^*(t-nT_i - \tau_i) \]

(30)

by using Poisson's sum formula we can obtain

\[ E[|y(t)|^2] = \sum_{i=1}^{p} \sigma_i^2 \gamma_i^2 \frac{1}{T_i} \sum_{m=-\infty}^{\infty} Z_{i,m} e^{j2\pi m(t-\tau_i)/T_i} \]

(31)

where

\[ Z_{i,m} = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_i(j\theta) H_i^{*} \left( -\frac{2\pi m}{T_i} - \theta \right) d\theta \]

(32)

and \( H_i(j\omega) \) is the frequency response of \( h_i(t) \).

Usually the pulse shaping filters are raised cosine filters with excess bandwidth less than 100%, that is, rolloff factor is less than unity [10]. Therefore \( h_i(t) \) is real and even and \( H_i(j\omega) \) is also real and even, which results that \( Z_{i,-m} = Z_{i,m}^* \) and \( Z_{i,m} \) is real. On the other hand, \( Z_{i,m} \) is zero for \( m \geq 2 \) since the rolloff factor is less than unity. Therefore, we can obtain

\[ E[r^2(t)] = 1/2 \cdot \left[ E[|y(t)|^2] + E[|w(t)|^2] \right] \]

\[ = \frac{1}{2} \sum_{i=1}^{p} \sigma_i^2 \gamma_i^2 \frac{1}{T_i} \sum_{m=-\infty}^{\infty} Z_{i,m} \cos \left( 2\pi (t-\tau_i)/T_i \right) \]

\[ + \frac{1}{2} \sum_{i=1}^{p} \sigma_i^2 \gamma_i^2 Z_{i,0} + \frac{1}{2} \sigma_w^2 \]

(33)

REFERENCES


