Distributed Spectrum Sharing in Cognitive Radio Networks - Game Theoretical View

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Abstract—Consumer communication networking allows connections among consumer devices for possible Ad Hoc or heterogeneous networking on licensed and unlicensed bands. To support maximum flexibility of consumer communication networks, cognitive radio (CR) technology by spectrum sharing can well serve the purpose. Realistic spectrum sharing for cognitive radio networks (CRN) shall be distributed and based on partially available information of spectrum sensing, due to a possible good number of cognitive radios and impossible to perfectly exchange channel availability information. However, existing study assumes either centralized or perfect spectrum sensing. In this paper, we pioneer explore spectrum sensing as a side information and take imperfection of spectrum sensing results into consideration. We therefore develop spectrum sharing algorithms for spectrum access strategy of CRs under two general scenarios: public spectrum information broadcast from the base station and private spectrum information via individual spectrum sensing. Under the public spectrum in-formation, CRs are aware of the strategy of its opponents and therefore game theory model reaches Nash equilibrium as a solution. On the other hand, CRs have to follow maximin criterion with only local spectrum information. Difference in the behaviors of CRs between the two scenarios is identified and the optimal spectrum access strategy is proposed accordingly for both cases. Numerical results demonstrate that the proposed algorithms work effectively and prevent the system from collision even in a large network.

I. INTRODUCTION

Consumer networks have been considered as emergence of the next generation networks, in which consumer devices with heterogeneous radio access technology are integrated to enhance the flexibility of the network. To coordinate communication among heterogeneous agents, the devices can share spectrum bands unoccupied by primary users. Cognitive radio (CR), a newly proposed concept based on the idea of software defined radio [1], makes the realization of opportunistic spectrum access possible [2].

Spectrum sharing can be either conducted via a centralized controller [3][4] or realized in distributed manners [5][6][7]. However, centralized spectrum sharing mechanism can be impractical with the drastic increment in spectrum demand, which leads to unacceptable computation complexity on the server side. On the other hand, distributed spectrum sharing together with consumer devices incorporated with CR capability provides a realistic spectrum sharing solution, in which CR users opportunistically sense the spectrum and make spectrum access decision individually. In this case, since the autonomic property of CRs is fully exploits in the distributed spectrum sharing, coordination and coexistence among the nodes has become a critical issue. Game theory introduces well-fitted models to describe interaction among such intelligent devices. In [5], CRs form coalitions that coordinate spectrum sharing cooperatively by performing a bargaining process. Nevertheless, incentive-based mechanism must be provided to make CRs share information with others; otherwise, CRs shall act in a selfish and non-cooperative way. Non-cooperative power allocation over the shared band is discussed in [6]. In [7], spectrum sharing is modeled as a non-cooperative market competition game. In all the previous works, however, spectrum knowledge is assumed to be public and available to every CR involved. However, realistic spectrum sharing shall be based on partial spectrum information due to limited sensing capability of CRs.

In this paper, we practically deal with the scenario, in which only partial spectrum information is available for individual CR that acts in a non-cooperative manner. We discuss the distributed medium access strategy with partial spectrum information under two different scenarios: public (i.e. spectrum information broadcast via spectrum agent) and private (i.e. spectrum information obtained via individual CR spectrum sensing) spectrum information. For the public spectrum information scenario, we identify the mixed strategy equilibrium under symmetric behavior of CRs, suggesting that the equilibrium depends on the number of CR in the network. For the private spectrum information obtained from individual sensing, CRs are differentiated by local spectrum knowledge. Symmetric behavior is no longer a reasonable assumption in this case. Therefore, maximin criterion is applied to design with respect to the worst channel access strategy of the opponents. We propose algorithms for both scenarios to determine spectrum access strategy and demonstrate that the algorithms prevent the system from collision in large network. The numerical results are presented in comparison with random and proportional channel selection. The proposed algorithms show superiority in system throughput over the other strategies in both scenarios.

II. SYSTEM MODEL AND ASSUMPTION

A. Terminology and Network Topology

The development of this paper is based on the network model illustrated in Fig. 1. Three types of communication devices are involved in the spectrum sharing procedures: Primary Mobile Station (PR-MS), Cognitive Radio Mobile Station (CR-MS), and Spectrum Agent (SA). A PR-MS is
a licensed node, which is able to transmit using the corresponding licensed band with top priority. On the other hand, a CR-MS is an unlicensed transmitter/receiver pair (CR-MS-Tx and CR-MS-Rx) with cognition capability to obtain spectrum information for spectrum access decision. An SA acts as both a spectrum manager and a spectrum information relay. From PR-MSs’ point of view, an SA gathers spectrum access requisition from PR-MSs and validate the packet transmission of PR-MS. From CR-MSs’ perspective, an SA broadcasts spectrum information and system parameters required for CR-MSs’ spectrum sharing strategies. In the network, we assume that there are $M$ PR-MSs and $K$ CR-MSs under the coverage of an SA. The $M$ PR-MSs correspond to a set of numbered licensed channels $M = \{1, \ldots, M\}$.

### B. Superframe Structure

The superframe is composed of several time slots defining the timing that different actions must be taken by PR-MSs and CR-MSs as shown in Fig. 2. Superframes are defined with equal time interval $[t_n, t_{n+1}]$, where $n \in \mathbb{Z}^+$ is the frame index. Timing knowledge is the common information among nodes in the network. Perfectly synchronous slotted-operation is assumed. In the beginning of each superframe, the SA accumulates spectrum access requisition from PR-MSs with packet arrival for $t_{req}$. Next, PR-MSs start to transmit at the end of the spectrum requisition for $t_{pac}$. At the same time, CR-MS-Txs initiate the spectrum sensing procedure for $t_s$ by either listen for the spectrum information broadcast from SA (Part A, Section III) or attain private spectrum information via spectrum sensing (Part B, Section III). To agree on the channel usage of a CR-MS pair, the CR-MS-Rx simply listens for the preamble of the corresponding CR-MS-Txs followed by the information indicating which channel to be used for data transmission. Finally, the CR-MSs perform data transmission for $t_{sac}$ till the end of the superframe.

### C. Decision Policy

In determining the channel selection and access strategy, spectrum information indicating the availability of the spectrum obtained in a superframe is used in CR-MS’s decision process. We denote the set of spectrum information attained over the channel set $M$ as $I_{k,n} = I_{m,k,n}$, where

$$I_{k,m,n} = \begin{cases} 1 & \text{if channel } m \text{ is available} \\ 0 & \text{if channel } m \text{ is occupied} \\ u_{k,m} & \text{if the status of channel } m \text{ is unknown} \end{cases}$$

The Bernoulli random variable $u_{k,m}$ with parameter $\theta_{k,m}$ indicates limited sensing capability of CR-MSs. In absence of the knowledge whether the PR-MS is transmitting, CR-MSs can make decisions solely based on the probability that the spectrum is available, i.e. $\theta_{k,m}$. We assume that the probability is common knowledge among CR-MSs. Thus, we discard the CR-MS index $k$ and denote the parameter as $b_m$.

A CR-MS ends up in one of the three possible outcomes at the end of the spectrum access phase: (i) the CR-MS successfully transmits $b_m$ bits via channel $m$ when it is the only user accessing the band (ii) collision occurs if either any other CR-MS transmits on the same band simultaneously or the band is occupied by the licensed PR-MS, leading to a collision cost denoted by $c$, where $c < 0$ (iii) the CR-MS does not transmit in the period, resulting in the payoff 0. We assume that two CR-MSs do not interfere with each other as long as they operate in different channels.

Each CR-MS determines a set of strategy profile, $S_k = \{p_{k,0}, p_{k,1}, \ldots, p_{k,M}\}$ indicating that the $k_{th}$ CR-MS accesses channel with probability $p_{k,m}$ for $m = 1, \ldots, M$ and does not access any channel with probability $p_{k,0}$. The decision is only applicable in a superframe and must be re-determined based on the spectrum information derived in other superframes.

The expected payoff $W_k(n)$ over the strategy profile of the $k_{th}$ CR-MS in $[t_n, t_{n+1}]$ is

$$W_k(n) = \frac{\sum_{m=1}^{M} p_{k,m} \left\{ c \left[ 1 - \prod_{j \neq k} (1 - p_{j,m}) \right] + b_m \right\}^{K} \prod_{j=1}^{M} (1 - p_{j,m})}{\prod_{j=1}^{K} (1 - p_{j,m})} + \frac{b_m \left\{ \prod_{j=1}^{K} (1 - p_{j,m}) \right\}^{K}}{\prod_{j=1}^{K} (1 - p_{j,m})} + (1 - \theta_m)c$$
III. OPTIMAL SPECTRUM SHARING STRATEGY

A. Spectrum Sharing with Public Spectrum Information

Symmetric behavior of CR-MSs is a critical assumption in public spectrum information scenario. The behaviors of CR-MSs are differentiated by the private spectrum sensing result. When spectrum information is public among CR-MSs, CR-MSs shall follow the same behavior. Thus we assume that the group strategy profile is \( S_k = S = \{p_0, p_1, \ldots, p_M\} \). The following fundamental theorem helps us to find out the equilibrium strategy profile:

**Theorem 1**: A mixed strategy profile \( \{p_0^*, p_1^*, \ldots, p_M^*\} \) is a mixed strategy equilibrium if and only if for each CR-MS: (i) The expected payoff corresponding to the strategy with \( p_m^* > 0 \) must be the same, given the strategy profile of the opponents. (ii) The expected payoff corresponding to the strategy with \( p_m^* = 0 \) must be less than or equal to those with \( p_m^* > 0 \), given the strategy profile of the opponents.

**Proof**: This is a straightforward extension from [8]. \( \square \)

Based on Theorem 1 and the assumption of symmetric behavior, we first consider the scenario with CRs and three bands as an example. The public spectrum information is \( 1_k(n) = \{u_1, u_2, u_3\} \). It is a general form of spectrum information since when \( \theta_m = 1 \), it means that the \( nth \) band is not occupied by the PR-MS and is occupied if \( \theta_m = 0 \). The mixed strategy profile can then be solved via the following equations along with constraints:

\[
\theta_m \left\{ b_m(1-p_m)^{K-1} + c[1-(1-p_m)^{K-1}] \right\} + c(1-\theta_m) = 0 \quad m = 1, 2, 3 \tag{2}
\]

where \( 0 < p_1, p_2, p_3 \) and \( p_1 + p_2 + p_3 = 1 \). The mixed strategy equilibrium can then be found by solving (2):

\[
p_m^* = 1 - \left( \frac{c}{(c-b_m)\theta_m} \right)^{K-1} \quad m = 1, 2, 3 \tag{3}
\]

\[
p_0^* = \left( \sum_{m=1}^{3} \left( \frac{c}{(c-b_m)\theta_m} \right)^{K-1} \right)^{-1} - 2 \tag{4}
\]

The mixed strategy equilibrium in (3) and (4) exists if and only if the following inequalities hold:

\[
\frac{c}{(c-b_m)\theta_m} < 1 \quad \forall m \tag{5}
\]

following inequalities hold:

\[
p_0^* = \left( \sum_{m=1}^{3} \left( \frac{c}{(c-b_m)\theta_m} \right)^{K-1} \right) > 2 \tag{6}
\]

meaning that a CR-MS never assign positive probability on channels bringing negative payoffs even when no other CRs access the channel simultaneously. Furthermore, if the inequality (5) holds, the term on the left side of inequality (6) is an increasing function of \( K \) bounded by 3. When the value of \( K \) is small relative to the number of available channel, the equilibrium with positive probability on not accessing any channel may not hold. Intuitively speaking, CR-MSs shall not be too pessimistic; instead, they could persist on accessing the channel to obtain positive payoff. The proposed algorithm below shows that some degree of persistency is allowed when fewer CR-MSs exist in the network.

**Proposition 2**: In the scenario with \( K \) users and \( M \) channels along with public spectrum information, the group behavior of the CR-MSs’ strategy profile \( S_k = S = \{p_0, p_1, \ldots, p_M\} \) under equilibrium is determined as:

1) For channels with \( \frac{c}{(c-b_m)\theta_m} > 1, p_m^* = 0 \).
2) Let the number of channel exclusive of those in 1) be \( M \). The strategy profile for those dissatisfies 1) is determined by:

\[
\text{If } \sum_{m=1}^{M} \left( \frac{c}{(c-b_m)\theta_m} \right)^{K-1} > M - 1,
\]

\[
p_m^* = 1 - \left( \frac{c}{(c-b_m)\theta_m} \right)^{K-1}.
\]

Else,

a) Set \( b_0 = 0 \), the payoff of not accessing channel.

b) Order the channel benefit in decreasing order exclusive of those satisfying 1).

c) Set \( M = 2 \).

d) Assign positive probability on the \( M \) channels with the largest benefits.

e) Solve for the equilibrium by Lemma 1 and check if the expected payoff is larger than that of the \( M + 1 \) channel. If yes, this is the equilibrium; otherwise, set \( M = M + 1 \) and repeat d) and e). \( \square \)

B. Spectrum Sharing with Private Spectrum Information

In this part, we consider a more realistic scenario, in which individual CR-MS makes decision solely depending on the private spectrum sensing information, since it is not always the case that an SA broadcast spectrum information to secondary users. Due to the fact that spectrum information of the opponents is not obtainable, a CR-MS is not able to reach a global equilibrium solution without the knowledge of its opponents’ channel access strategies. The design philosophy for each CR turns out to be the maximin criterion, i.e. A CR-MS determines the strategy profile such that

\[
S_k^* = \arg\max_{S_k} \left( \min_{S_{-k}} W_k(S_k, S_{-k}) \right) \tag{7}
\]

where \( S_{-k} \) is the strategy profile for all CR-MSs with index \( j \neq k \).
We start by exploring the two-user and two-channel case. Let the spectrum information obtained by CR1 be $1_k(n) = \{1, 1\}$. Assume that the strategy profile of CR1 is $S_k = \{0, p_{1,1}, p_{1,2}\}$. To find out the maximin strategy, we refer to Fig. 3, in which the horizontal and vertical axis represent the probability that CR1 accesses band 1 and the payoff of CR1, respectively. The labeled line segment shows the payoff of CR1 as a function of $p_{1,1}$ when different pure strategy is selected by its opponent, CR2. The label $i$ ($i = 1, 2$) indicates which numbered band is selected by CR2 with probability 1, and the label "X" means that CR2 does not transmit with probability 1. The bold line segment represents the worst payoff that CR1 can achieve as a function of $p_{1,1}$. It is obvious that the maximin strategy is $p_{1,1}^*$, at which the worst payoff function is maximized and "equalized" regardless the strategy chosen by CR2. Thus, we have the maximin strategy profile, which is also called an equalizer rule, for CR1:

$$S_k = (0, \frac{b_2 - c}{b_1 + b_2 - 2c}, \frac{b_1 - c}{b_1 + b_2 - 2c}) \quad (8)$$

Note that the strategy profile in (8) is feasible if and only if it brings positive payoff function. Otherwise, CR1 shall choose not to access any channel with probability 1. To generalize for a scenario with $K$ users and $M$ channels, we begin with the following lemmas:

**Lemma 3**: A CR-MS assigns probability 0 on those channels with $I_{k,m}(n) = 0$ or on those with $(I_{k,m}(n) = u_k, m \& (b_m - c)\theta_m + c < 0)$.

**Lemma 3** is a procedure of eliminating dominated strategy as explained in (5). Thus in the following, when referring to "$M$ channels," we excludes those satisfying conditions in Lemma 3.

Furthermore, we can think of the other $K - 1$ CRs as a single opponent. The worst that the single opponent can harm the CR is to occupy at most $K - 1$ bands, which we term as "worst case combination" in the following. Therefore, any strategy of the environment that occupies less than $K - 1$ bands is not of our concern in finding maximin strategy. The following Lemma is another example:

**Lemma 4**: In the scenario with $K$ users and $M$ channels, if $M = K$ for the CR-MS, the equalizer rule is maximin if the expected payoff is greater than 0. The maximin strategy is

$$p_m^* = \sum_{j=1}^M (b_j - c)\theta_j^p$$

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$$\sum_{j=1}^M (b_j - c)\theta_j^p$$

where $\theta_j^p = \theta_j$ if channel $j$ is in unknown status and $\theta_j^p = 1$ if channel $j$ is not occupied.

**Proof**: For the equalizer rule $\{p_1^*, p_2^*, ..., p_M^*\}$, the deviation version can be written as $\{p_1^* + \varepsilon_1, p_2^* + \varepsilon_2, ..., p_M^* + \varepsilon_M\}$ with $\sum_{m=1}^M \varepsilon_m = 0$. The strategy profile achieves smaller payoff than the equalized one under the case that the channels with $\varepsilon_m > 0$ are occupied. The result in (9) is simply an extended version of (8).

The next Lemma shows the role of the equalizer rule in finding the maximin strategy.

**Lemma 5**: In the scenario with $K$ users and $M$ channels, a profile that does not equalize the opponents’ pure strategy worst case combination cannot be a maximin strategy.

**Proof**: Consider a small deviation from the equalizer rule, say $\{p_1^* + \varepsilon_1, p_2^* + \varepsilon_2, ..., p_M^* + \varepsilon_M\}$, in which channel $j$ is the one that the opponents access in the profile that brings least payoff, and channel $k$ is the one that is not occupied by other CR-MSs. We can find $\varepsilon > 0$ which is small enough such that the order of the payoff for the pure strategy profile of the opponents remains. Thus, the worst case pure strategy is improved.

In that case, the deviation from equalizer rule cannot be maximin strategy.

In summary, to find out the maximin strategy profile in the scenario with $K$ users and $M$ channels, a CR-MS should refer to the equalizer rule. Nevertheless, it might be too pessimistic to equalize over all the $M$ channels especially when the cost of collision is low. Certain degree of persistency is allowed in this case. Based on the above Lemmas, we propose the following algorithm to find out the maximin strategy profile.

**Proposition 6**: In the scenario with $K$ users and $M$ channels, where $K < M$, we find the maximin strategy profile following the procedures below:

1) Select $m = K$ channels with the largest benefits
2) Find equalizer strategy profile over the channels selected in 1), and assign probability 0 on those not selected.
3) Calculate the equalized payoff in 2)
4) Set $m = K + 1$ and repeat 1)-4) till $m = M + 1$

The maximin strategy profile is determined by choosing the profile bringing the largest equalized payoff calculated in 3). If the largest payoff is smaller than 0, choose not to access with probability 1.
IV. NUMERICAL EXAMPLE AND SIMULATION RESULTS

We assume that there are 10 unoccupied channels. The benefits are randomly generated from $(0,10]$. $\theta_m$’s are randomly generated from $[0,1]$. Collision payoff is assumed to be -1. Total benefit is defined as the summation of the channel benefit for successful transmission plus the cost for collision. Simulation results are averaged over $10^5$ times iteration. The result is compared with random channel assignment and proportional mixed strategy assignment. For the random assignment, CRs access each channel with equal probability; whereas for the proportional assignment, CRs access channel $m$ with probability $b_m / \sum_{j=1}^{M} b_j$.

When public spectrum information is available, it can be clearly seen in Fig. 4 that the proposed algorithm outperforms the random channel assignment in the region where the number of CR-MSs is smaller than 5 (an increment in benefit by 4) and the number of CR-MSs is larger than 18 (an increment in benefit ranging from 0 to 15). It is also superior to the proportional assignment when the number of CR-MSs is larger than 11. The reason for the outperformance in the proposed algorithm as the number of CRs increases is that the probability that a CR-MS does not access any channel increases and enforces the expected payoff of individual CR-MS to 0. For a small number of CR-MS, the proposed mechanism makes CR-MSs persistent on the channel with highest benefits, assuring higher benefit than random assignment by preserving the merit of proportional assignment. Although the inefficiency of Nash equilibrium due to non-cooperative is presented in the region [5,18], the proposed algorithm successfully prevents the system from collision when a large number of CR-MSs coexists in the network. The total benefit is held at 8 for the proposed algorithm, which is a tremendous increment compared with the other two strategies.

The performance of the Proposition 6 based on private spectrum sensing knowledge is shown Fig. 5. It can be seen that the maximin strategy outperforms the other two strategy profiles in the total throughput. The reason can be the fact that the proposed algorithm preserves the merit of proportional access and uniform assignment. When the number of CR nodes involved are small, the proposed strategy makes CR persistent on certain channels with high benefit. When a large number of CRs are presented, the strategy becomes conservative by assigning positive probability on more channels, leading to the decreased probability of packet collision and therefore the increment in system throughput.

V. CONCLUSION

Spectrum sharing, an essential technique in organizing communication among heterogeneous devices in consumer networks, has been proved to increase spectrum utilization in previous efforts. While distributed spectrum sharing alleviate possible tremendous computation at a centralized controller in centralized scheme, the total number of CRs accessing the channels can be unlimited in absence of the centralized controller, resulting in frequent packet collision and degradation in system throughput. To make CRs self-coordinate in the distributed scheme, we propose spectrum sharing mechanism based on the non-cooperative game theoretic analysis. Partially available spectrum information is investigated in our work. Algorithms for scenarios in both public and local spectrum knowledge are proposed. CRs perform randomized decision rule in determining the channel to be used. With the public spectrum knowledge, CRs act in a symmetric manner and seek for a mixed strategy equilibrium. On the other hand, when only private sensing information available, CRs follow a maximin criterion. Simulation results for both scenarios show that the proposed mechanism, which provide a self-enforcing solution to coordinate multiple CRs, ensure the system throughput even in large network in comparison with proportional and random selection strategy.

REFERENCES