Percolation-based Connectivity of Multiple Cooperative Cognitive Radio Ad Hoc Networks

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Abstract—In cognitive radio networking, the signal reception quality of a secondary user degrades due to the interference from multiple heterogeneous primary networks, and also the transmission activity of a secondary user is constrained by its interference to the primary networks. It is difficult to ensure the 1-connectivity of the secondary network, so we here characterize the connectivity of the secondary network from a percolation-based perspective. On the other hand, since there may exist multiple heterogeneous secondary networks with different radio access technologies, such secondary networks may be treated as one secondary network via proper cooperation. In this paper, we investigate the connectivity of such a cooperative secondary network, under which each secondary network’s user may have other secondary networks’ users acting as relays. The connectivity of this cooperative secondary network is characterized in terms of percolation threshold, from which the benefits of cooperation are justified. For example, while a noncooperative secondary network does not percolate, percolation may occur in the cooperative secondary network; or when a noncooperative secondary network percolates, less energy may be used to sustain the same level of connectivity in the cooperative secondary network.

I. INTRODUCTION

Cognitive radio networking has emerged as a promising technology to enhance spectrum utilization by sensing the spectrum and opportunistically accessing the spectrum of primary (licensed) systems. The spectrum sharing model can be practically divided into two major approaches: interweave and underlay [1]. This paper mainly focuses on the underlay approach, under which the primary users (PUs) and secondary users (SUs) transmit at the same time while SUs are considered of lower priority in spectrum access. Their interference to PUs should be limited by an interference temperature [1] indicating the maximum tolerable interference power. It is important to understand the connectivity of the secondary network so that proper network operations can be performed. However, different from a homogeneous network user, the signal reception quality of an SU further degrades due to the inter-system interference from PUs, and also the transmission activity of an SU is constrained by its interference to PUs. It is much harder to ensure the connectivity of the secondary network. As a result, if multiple secondary networks can cooperate with each other by having other secondary networks’ nodes acting as relays, connectivity may be improved.

Connectivity of a homogeneous wireless ad hoc network has been widely studied in different aspects. In [2], 1-connectivity (i.e., a path exists between any pair of nodes) of a network is investigated. However, in many applications such as sensor networks and mobile ad hoc networks, 1-connectivity is not required (or is difficult to be maintained), and a weaker connectivity from a percolation-based perspective is more suitable. For an infinite network with density $\lambda$ and transmission range $r$, the network percolates in which an infinite connected component appears when the average degree of a network node is above a certain number (termed percolation threshold $\beta_{th}$), i.e., $\lambda \pi r^2 \geq \beta_{th}$. Otherwise, the network does not percolate and consists of many isolated finite clusters. In [3], [4], the impact of fading on the percolation threshold of a wireless ad hoc network was investigated. The effect of self co-channel interference was considered in [5]. Percolation threshold of a cooperative wireless ad hoc network was addressed in [6], under which nodes may form a group to perform distributed beamforming so that signals are coherently combined at some far-away destinations. Note that all the above works were focused on connectivity of a homogeneous ad hoc network.

Connectivity of heterogeneous wireless networks, specifically cognitive radio networks, has been recently developed in [7]–[9]. In [7], the connectivity of a secondary network is parameterized by the tuple $(\lambda_{PT}, \lambda_{SU})$ under path loss channel model, where $\lambda_{PT}$ is the density of primary transmitters (PTs) and $\lambda_{SU}$ is the density of SUs. Percolation of the secondary network occurs only when $\lambda_{PT}$ is below some threshold and $\lambda_{SU}$ is above some threshold. Another parameterization of the secondary network $(\lambda_{SU}, r_{SU})$ is developed in [8] under Rayleigh fading channel model, where $r_{SU}$ is the transmission range of an SU with an outage constraint. The temporal activity of a spectrum opportunity is further captured in [9]. In spite of above efforts, the connectivity of cooperative secondary network has not been investigated.

In this paper, we therefore study the connectivity of cooperative secondary network from a percolation-based perspective. To our knowledge, it is the first framework to jointly consider the effects of cooperation (among multiple secondary networks), interference (from multiple primary networks), heterogeneity (in different priorities of spectrum access), and general fading on the percolation threshold. Our framework can be summarized into four steps. In the first step, we derive the degree distribution of an SU sustaining interference from

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multiple heterogeneous primary networks under general fading of the desired signal and interfering signals. In the second step, we obtain the active probability of an SU. An SU can transmit (or be active) only when its interference power to each primary receiver (PR) in the primary networks is below the interference temperature. In the third step, with both the degree distribution and the active probability of an SU, the connectivity of a single secondary network in percolation sense is derived by using the site-percolation model [10]. The site-percolation model describes the percolation threshold of a network (of some degree distribution) with random node failures. A random node failure has an analogy with the deactivation of an SU. In the fourth step, we study the connectivity of cooperative secondary network in percolation sense, under which each secondary network’s node may have other secondary networks’ nodes acting as relays. The benefits of cooperation among multiple secondary networks are analytically justified in terms of percolation threshold.

II. NETWORK MODEL

We consider a secondary network (denoted as $SN_0$), in which the spatial distribution of SUs is assumed to follow a homogeneous Poisson point process (PPP) with density $\lambda$. Let $\Phi = \{X_k\}$ denote the locations of the SUs and let $P$ denote the transmit power of an SU. $SN_0$ coexists with $N$ heterogeneous primary networks (denoted as $PN_i$, $1 \leq i \leq N$), where the spatial distribution of PTs (resp. PRs) in $PN_i$ is assumed to follow a PPP with density $\mu_i^{PT}$ (resp. $\mu_i^{PR}$).\(^1\) The locations of the PTs in $PN_i$ are denoted as $\Psi_i^{PT} = \{Y^i_k\}$ with transmit power $P_i$, and the locations of the PRs in $PN_i$ are denoted as $\Psi_i^{PR} = \{Z^i_k\}$ with interference temperature $\tau_i$. The interference from the PTs in $PN_i$ to a typical SU (a reference SU located at the origin) is $i.i.d.$ $\Psi_i^{PT}$. $G_{Y_i}^k$ denotes the channel power gain of the interfering link from the PT at $Y^i_k$ to the typical SU, which is assumed to be independently drawn according to some distribution $f_{G_{Y_i}^k}$, i.e., the distribution function $f_{G_{Y_i}^k}(x) = f_{G_{Y_i}^k}(x), \forall k, \|Y^i_k\|$ is the distance between the PT at $Y^i_k$ and the typical SU, and $\alpha$ is the path loss exponent. All packet transmissions of the networks are assumed to be slotted and synchronized. Also, since $SN_0$ operates in an interference-limited environment, we ignore the background noise.

As a result, neighbors of a typical SU of $SN_0$ are resulted from independently\(^3\) thinning of each SU $X_k \in \Phi$ with probability $\mathbb{P} \left( \log \left( 1 + \frac{G_{X_k}^i \|X_k\|^{-\alpha}}{\sum_{i=1}^{\infty} G_{Y_i}^k \|Y_i\|^{-\alpha}} \right) \geq R \right)$, where $G_{X_k}$ denotes the channel power gain of the desired link from the typical SU to the SU at $X_k$, which is assumed to be independently drawn according to some distribution $f_{G_{X_k}}$, i.e., the distribution function $f_{G_{X_k}}(x) = f_{G_{X_k}}(x), \forall k, \|X_k\|$ denotes the distance between the typical SU and the SU at $X_k$, and $R$ denotes the information rate. A scenario is shown in Fig. 1(a). By the mapping theorem [11], the number of neighbors of a typical SU of $SN_0$ is Poisson distributed with mean $\beta = \int_0^\infty \lambda \mathbb{P} \left( G_{X_k} \geq \left( \frac{2^{R-1}(\sum_{i=1}^N P_i)^{-\alpha}}{P_{PR}^{-\alpha}} \right) \right) 2\pi r dr$, which is further computed in the next subsection.

A. Degree distribution of an SU

For the purpose of the following derivations, we let $z \triangleq \frac{g_{X_k} P_i r_i}{2^{R-1}(\sum_{i=1}^N P_i)^{-\alpha}}$ ($g_{X_k}$ is a realization of $G_{X_k}$) and define the set $\Xi_i^{PT} = \{Y^i_k \in \Psi_i^{PT} : G_{Y_i}^k \|Y_i\|^{-\alpha} > z\}$, which consists of dominating interferers from $PN_i$. A dominating interferer is a PT that causes outage at a typical SU of $SN_0$ with its sole interference contribution. By the mapping theorem, the number of PTs in $\Xi_i^{PT}$ is Poisson distributed with mean $\int_0^\infty \mu_i^{PT} \mathbb{P} \left( G_{Y_i}^k P_i r_i^{-\alpha} > z \right) 2\pi r dr$.

\[
\begin{align*}
&= \mathbb{E}_{G_{Y_i}^k} \left[ \int_0^\infty \mu_i^{PT} 1 \left( r_i < \left( G_{Y_i}^k z_i^{-1} \right)^{\frac{1}{\alpha}} \right) 2\pi r dr \right] \\
&= \mathbb{E}_{G_{Y_i}^k} \left[ \mu_i^{PT} \pi \left( G_{Y_i}^k z_i^{-1} \right)^{\frac{1}{\alpha}} \right] = \pi \mu_i^{PT} \mathbb{E}_{G_{Y_i}^k} [G_{Y_i}^k]^{-\frac{1}{\alpha}} \\
&= \pi \mu_i^{PT} \mathbb{E}_{G_{Y_i}^k} [G_{Y_i}^k] P_i^{\frac{1}{\alpha}} (2^{R-1} - 1)^{\frac{-1}{\alpha}} g_{X_k}^{-\frac{1}{\alpha}}, \quad (1)
\end{align*}
\]

\(^1\)For example, in $PN_i$, each PT has a dedicated PR at distance $d_i$ away with an arbitrary direction so that the PRs also form a PPP with density $\mu_i^{PR} = \mu_i^{PT}$.

\(^2\)By the stationary characteristic of homogeneous PPP [11], the statistics measured by the typical node is representative for all other nodes. Therefore, we focus our discussions on the typical node. A typical node may be a transmitter or a receiver depending on the context.

\(^3\)The spatial correlation of interference observed at different nodes in $\Phi$ is ignored due to the assumption that the channel gains in different pairs of nodes are i.i.d. [12].
where $\delta = \frac{2}{\pi} \text{ and } 1(\cdot)$ is the indicator function. The probability that no PTs exist in $\Xi_i^{PT}$ is
\[
\mathbb{P}(\Xi_i^{PT} = \emptyset) = \exp\left(-\sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta]\right) \left(2^{R-1} - 1 \right)^{\delta} g_{X}^{-\delta} \tag{2}\]

We further define the set $\Xi_i^{PT} = \bigcup_{i=1}^{N} \Xi_i^{PT}$ consisting of dominating interferers from all $PN_i, 1 \leq i \leq N$.

Now, $\beta$ can be evaluated as
\[
\beta = \int_0^\infty \lambda \mathbb{P} \left( G_X \geq \frac{(2^{R-1} - 1) \sum_{i=1}^{N} I_i}{P_{\gamma}^\alpha} \right) 2\pi r \text{d}r
\]
\[
= \int_0^\infty \lambda \mathbb{E}_{G_X} \left[ \mathbb{P} \left( \sum_{i=1}^{N} I_i \leq \frac{G_X P_{\gamma}^\alpha}{2^{R-1}} \right | G_X = g_X \right) \right] 2\pi r \text{d}r
\]
\[
= \mathbb{E}_{G_X} \left[ \int_0^\infty \lambda \mathbb{P} \left( \sum_{i=1}^{N} I_i \leq z \right) 2\pi r \text{d}r \right]
\]
\[
< \mathbb{E}_{G_X} \left[ \int_0^\infty \lambda \exp \left\{ - \sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right\} \times \pi r^2 \left( 2^{R-1} - 1 \right)^{\delta} g_{X}^{-\delta} \right] 2\pi r \text{d}r \]
\[
= \lambda \mathbb{E}_{G_X} \left[ \exp \left\{ - \sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right\} \times \pi r^2 \left( 2^{R-1} - 1 \right)^{\delta} g_{X}^{-\delta} \right] \left( \sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right) \left( 2^{R-1} - 1 \right)^{\delta} g_{X}^{-\delta}
\]
\[
\equiv \beta^{\text{upper}} \tag{3}
\]

where (a) follows by (2). Note that $\beta^{\text{upper}}$ is the upper bound of $\beta$. We conclude that the degree distribution of an SU sustaining interference from multiple heterogeneous primary networks (denoted as $p_k$) is Poisson with mean $\beta$, i.e.,
\[
p_k = \frac{\exp(-\beta) \beta^k}{k!}, \quad k = 0, 1, 2, \ldots \tag{4}
\]

As a remark, when the desired link is Rayleigh faded, i.e., its channel power gain $G_X$ is exponentially distributed (with unit mean, without loss of generality), we can obtain the exact $\beta$ in a closed-form. From [13], the Laplace transform of the interference $I_i$ (a 2D Poisson shot noise process) is
\[
\mathcal{L}_{I_i}(s) = \mathbb{E}_{I_i}[e^{-s I_i}] = \exp\left(-\mu_i^{PT} \pi P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \Gamma(1-\delta) s^\delta \right) \tag{5}
\]

where $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ is the Gamma function. Let $\theta = \frac{2^{R-1}}{\pi P_{\gamma}^\alpha}$, we have
\[
\beta = \int_0^\infty \lambda \mathbb{P} \left( G_X \geq \frac{(2^{R-1} - 1) \sum_{i=1}^{N} I_i}{P_{\gamma}^\alpha} \right) 2\pi r \text{d}r
\]
\[
\equiv \int_0^\infty \lambda \mathbb{E}_{I_1, \ldots, I_N} \left[ \exp \left( -\theta \sum_{i=1}^{N} I_i \right) \right] 2\pi r \text{d}r
\]
\[
= \int_0^\infty \lambda \prod_{i=1}^{N} \mathbb{E}_{I_i} \left[ \exp \left( -\theta I_i \right) \right] 2\pi r \text{d}r
\]
\[
= \int_0^\infty \lambda \exp \left( - \sum_{i=1}^{N} \mu_i^{PT} \pi P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right) \times \pi r^2 \left( 2^{R-1} - 1 \right)^{\delta} P_{\gamma}^\alpha \Gamma(1-\delta) 2\pi r \text{d}r
\]
\[
= \lambda \mathbb{E}_{G_X} \left[ \exp \left\{ - \sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right\} \times \pi r^2 \left( 2^{R-1} - 1 \right)^{\delta} \right] \left( \sum_{i=1}^{N} \mu_i^{PT} P_i^{\delta} \mathbb{E}[G_{Y_i}^\delta] \right) \left( 2^{R-1} - 1 \right)^{\delta} g_{X}^{-\delta}
\]
\[
\equiv \beta^{\text{upper}}, \tag{6}
\]

where (a) follows $\mathbb{P}(G_X > x) = e^{-x}$, and (b) follows by (5). B. Active probability of an SU

The interference power from a typical SU of $SN_0$ to the PR at $Z_k \in \Psi_i^{PR}$ is $G_{Z_k} P_i |Z_k|^{-\alpha}$, where $G_{Z_k} \sim N \sim N$ denotes the channel power gain of the interfering link from the typical SU to the PR at $Z_k$, which is assumed to be independently drawn from $f_{G_{Z_k}}(x) = f_{G_x}(x), \forall k$. $|Z_k|$ denotes the distance between the typical SU and the PR at $Z_k$. A scenario is shown in Fig. 1(a). We define the set $\Xi_i^{PR} = \{ Z_k \in \Psi_i^{PR} : G_{Z_k} P_i |Z_k|^{-\alpha} > \tau_i \}$, which consists of PRs in $PN_i$, whose received interference power from the typical SU exceeds the interference temperature $\tau_i$.

From mapping theorem, the number of PRs in $\Xi_i^{PR}$ is Poisson distributed with mean $\nu_i$
\[
\nu_i = \int_0^\infty \mu_i^{PT} \mathbb{P} \left( G_Z, P_i R^{-\alpha} > \tau_i \right) 2\pi r \text{d}r = \mu_i^{PT} \pi P_i^{\delta} \mathbb{E}[G_Z^\delta] \tau_i^{-\delta} \tag{7}
\]

An SU can be active only when there are no PRs in $\Xi_i^{PR}, \forall i, 1 \leq i \leq N$. Thus, the active probability of an SU (denoted as $p^\alpha$) can be computed as
\[
p^\alpha = \prod_{i=1}^{N} \mathbb{P}(\Xi_i^{PR} = \emptyset) = \prod_{i=1}^{N} \exp(-\nu_i)
\]
\[
\equiv \exp \left\{ - \left( \sum_{i=1}^{N} \nu_i P_i^{\delta} \mathbb{E}[G_{Z_i}^\delta] \tau_i^{-\delta} \right) \pi P_i^{\delta} \right\}, \tag{8}
\]

where (a) follows (7).

C. Mapping to site-percolation model

From Section II-A and Section II-B, we obtain the degree distribution of an SU $p_k$, which is Poisson with mean $\beta$, and
the active probability of an SU $p^A$. Now, we are able to derive the percolation threshold of the secondary network $SN_0$ by using the site percolation model [10].

We define the generating function of the degree distribution of a randomly selected SU as $F_0(x) = \sum_{k=0}^{\infty} p_k x^k = \exp(\beta(x - 1))$. The excess degree\(^4\) distribution of an SU at the end of a randomly selected link is defined as $q_k = (k + 1)p_{k+1}/\sum_k k p_k = (k + 1)p_{k+1}/\beta$, which is proportion to $k p_k$ since it is likely to arrive at a node that has a higher degree by following a randomly selected link [14]. The generating function of $q_k$ is defined as $F_1(x) = \sum_{k=0}^{\infty} q_k x^k$, and we can easily verify that $F_1(x) = F_0(x)/\beta = \exp(\beta(x - 1))$. Moreover, the generating function of the total number of active SUs reachable by following a randomly selected link (resp. SU) is defined as $H_1(x)$ (resp. $H_0(x)$), which satisfies the self-consistency relation $H_1(x) = 1 - p^A + p^A x^{\sum_{k=0}^{\infty} q_k (H_1(x))^k}$ (resp. $H_0(x) = 1 - p^A + p^A x^{\sum_{k=0}^{\infty} p_k (H_1(x))^k}$). An illustration is shown in Fig. 2(a). Consequently, the average total number of active SUs reachable from a randomly selected SU (or the mean size of a cluster of connected and active SUs) is $H'_1(1) = p^A [1 + p^A F_1(1)/(1 - p^A F_1(1))]$, which blows up when $1 - p^A F_1(1) = 0$ indicating the formation of an infinite connected component. Here in the Poisson case, it becomes $1 - p^A \beta = 0$ and the percolation threshold is $\beta_{th} = 1/p^A$. That is, the secondary network percolates when $\beta \geq \beta_{th} = 1/p^A$.

Note that the above formulation ignores the effect of spatial dependency (i.e., two of the node’s neighbors are likely neighbors themselves) before network percolates. The actual percolation threshold is offset with a small constant depending on the fading distribution. In the case without fading, the network is the so-called random geometric graph with $p^A \beta_{th} \approx 4.52$, while channel fading produces spread-out connections and weakens the spatial dependency, which reduces the percolation threshold [3], [4]. In our analysis, the spatial dependency may be further reduced due to the randomness from the interference. We here follow the approaches in [10], [14] to provide benchmark comparisons of the percolation thresholds of a noncooperative secondary network and the cooperative one.

### III. Cooperative secondary network

As discussed in Section II-C, when the mean degree of a secondary network node is lower than the percolation threshold $\beta_{th}$, the secondary network does not percolate. However, when different secondary networks cooperate with each other such that each secondary network’s node may have other secondary networks’ nodes acting as relays, percolation may be facilitated. We focus on cooperation between two heterogeneous secondary networks coexisting with $N$ heterogeneous primary networks. Suppose that there are two secondary networks (denoted as $SN_A$ and $SN_B$), respectively with node density $\lambda_A$ and $\lambda_B$, with nodes’ locations $\Phi_A = \{X^A_k\}$ and $\Phi_B = \{X^B_k\}$, with transmit power $P_A$ and $P_B$, and with information rate $R_A$ and $R_B$. The channel power gain of the link between a transmitting node in $SN_x$, $x \in \{A,B\}$ and a receiving node in $SN_y$, $y \in \{A,B\}$ is assumed to be drawn according to some distribution $f_{G_{x,y}}$. In addition, the channel power gain of the interfering link from a PT in $PN_i$, $1 \leq i \leq N$ to a receiving node in $SN_x$, $x \in \{A,B\}$ is assumed to be drawn according to some distribution $f_{G_{i,x}}$. A scenario is shown in Fig. 1(b). Following (3), we can evaluate the average number of neighbors in $SN_y$, $y \in \{A,B\}$ of a typical node of $SN_x$, $x \in \{A,B\}$ (denoted as $\beta_{x,y}$).

We take $\beta_{A,B}$ as an example. With cooperation, a node in $SN_B$ at distance $r$ listens and decodes the message from a typical node of $SN_A$, and becomes the typical node’s neighbor with probability

$$\mathbb{P}\left[ \log \left( 1 + \frac{G_{X_A,B} P_A R^{-\alpha}}{\sum_{i=1}^{\infty} I_i} \right) \geq R_A \right].$$

(9)

As a result, neighbors in $SN_B$ of a typical node of $SN_A$ are resulted from independently thinning of each node $X^B_i \in \Phi_B$ with probability $\mathbb{P}\left[ G_{X_A,B} \geq \frac{(2R_A - 1)(\sum_{i=1}^{\infty} I_i)}{P_A (\sum_{i=1}^{\infty} I_i)} \right]$. The number of neighbors in $SN_B$ of the typical node of $SN_A$ is thus Poisson distributed with mean $\beta_{A,B}$,

$$\beta_{A,B} = \int_{0}^{\infty} \lambda_B \mathbb{P}\left[ G_{X_A,B} \geq \frac{(2R_A - 1)(\sum_{i=1}^{\infty} I_i)}{P_A R^{-\alpha}} \right] 2\pi r dr,$$

(10)

$$\lambda_B P_A R^{-\alpha} \mathbb{E}[G^\delta_{X_A,B}] \left( \sum_{i=1}^{\infty} \mu_i P_i R^{\alpha} \mathbb{E}[G^\delta_{Y_i,B}] \right) (2R_A - 1)^{\delta}.$$  

Similarly, we can find the average number of neighbors in $SN_A$ of a typical node of $SN_B$, i.e., $\beta_{B,A}$,

$$\beta_{B,A} < \int_{0}^{\infty} \lambda_A \mathbb{P}\left[ G_{Y_A,B}^{\delta} \right] \left( \sum_{i=1}^{\infty} \mu_i P_i R^{\alpha} \mathbb{E}[G^\delta_{Y_i,A}] \right) (2R_B - 1)^{\delta}.$$  

(11)

Note that $\beta_{A,B}$ and $\beta_{B,A}$ are different. Also, $\beta_{A,A}$ and $\beta_{B,B}$ can be computed similarly.

Moreover, the active probability of an SU in $SN_A$ ($SN_B$) is denoted as $p^A_A$ ($p^A_B$). We assume that the channel power gain of the interfering link from a transmitting node in $SN_x$, $x \in \{A,B\}$ to a PR in $PN_i$, $1 \leq i \leq N$ is drawn according to some distribution $f_{G_{x,i}}$. A scenario is shown in Fig. 1(b). From (8), $p^A_A$ ($p^A_B$) can be computed as

$$p^A_A = \exp\left( - \sum_{i=1}^{N} \mu_i P_i R^{\alpha} \mathbb{E}[G^\delta_{Z_{A,i}}] \tau_i^{\delta} \right) \pi P_A,$$

$$p^A_B = \exp\left( - \sum_{i=1}^{N} \mu_i P_i R^{\alpha} \mathbb{E}[G^\delta_{Z_{B,i}}] \tau_i^{\delta} \right) \pi P_B.$$  

(12)

To obtain the percolation threshold of the cooperative secondary network, we here follow the development described in Section II-C. First, the degree distribution of a randomly selected SU in $SN_A$ (resp. $SN_B$) is denoted as $p^k_{A,k_B}$ (resp.
Third, the generating function of the total number of active SUs reachable by following a randomly selected link, which first arrives at an SU of \( S_N \) (resp. \( SN_B \)), is defined as \( H^{(1)}_I(x) \) (resp. \( H^{(2)}_B(x) \)). We have the self-consistency relations

\[
H^{(1)}_I(x) = 1 - p^A_A + p^A_B x \sum_{k_A,k_B=0}^{\infty} q^A_{k_A,k_B} [H^{(1)}_I(x)]^{k_A} [H^{(2)}_B(x)]^{k_B}
\]

\[
= 1 - p^A_A + p^A_B x F_1^A(H^{(1)}_I(x), H^{(2)}_B(x));
\]

\[
H^{(2)}_B(x) = 1 - p^B_B + p^B_B x \sum_{k_A,k_B=0}^{\infty} q^B_{k_A,k_B} [H^{(1)}_I(x)]^{k_A} [H^{(2)}_B(x)]^{k_B}
\]

\[
= 1 - p^B_B + p^B_B x F_1^B(H^{(1)}_I(x), H^{(2)}_B(x)).
\]

An illustration is shown in Fig. 2(b).

Fourth, the generating function of the total number of active SUs reachable by following a randomly selected SU of \( S_N \) (resp. \( SN_B \)) is defined as \( H^{(0)}_x \) (resp. \( H^{(0)}_B(x) \)), which can be computed as

\[
H^{(0)}_x = 1 - p^A_A + p^A_B x \sum_{k_A,k_B=0}^{\infty} q^A_{k_A,k_B} [H^{(1)}_I(x)]^{k_A} [H^{(2)}_B(x)]^{k_B}
\]

\[
= 1 - p^A_A + p^A_B x F_0^A(H^{(1)}_I(x), H^{(2)}_B(x));
\]

\[
H^{(0)}_B(x) = 1 - p^B_B + p^B_B x \sum_{k_A,k_B=0}^{\infty} q^B_{k_A,k_B} [H^{(1)}_I(x)]^{k_A} [H^{(2)}_B(x)]^{k_B}
\]

\[
= 1 - p^B_B + p^B_B x F_0^B(H^{(1)}_I(x), H^{(2)}_B(x)).
\]

Finally, from (18), the average total number of active SUs (or the mean component size of a cluster of connected and active SUs) reachable from an SU of \( S_N \) (resp. \( SN_B \)) is \( \partial_x H^{(0)}_x \) (resp. \( \partial_x H^{(0)}_B(x) \)). We have

\[
\begin{align*}
H^{(0)}_x(1) & = \left[ H^{(0)}_x(1) \right]^{(a)} \left[ H^{(0)}_B(1) \right]^{(b)} \\
& = \left[ p^A_A \right]^{(a)} \left[ p^B_B \right]^{(b)} + \left[ p^A_B \beta_{AA} p^B_A \beta_{AB} \beta_{AA} \right] H^{(1)}_I(1) \left[ H^{(2)}_B(1) \right]^{(b)} + \left[ p^B_A \beta_{BA} p^A_B \beta_{BA} \right] \left[ 1 - p^A_A \beta_{AA} - p^A_B \beta_{AB} \right] \left[ 1 - p^B_B \beta_{BB} - 1 - p^B_B \beta_{BB} \right] \left[ -1 \right] p^B_A \beta_{BA} \beta_{BB} \left[ -1 \right] p^A_B \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right]
\end{align*}
\]

where (a) follows by (18) and (14), and (b) follows by (17), (15), and (16).

Let \( C \approx \left[ 1 - p^B_B \beta_{AA} - p^A_B \beta_{AB} \right] \left[ 1 - p^B_B \beta_{BB} - 1 - p^B_B \beta_{BB} \right] \left[ -1 \right] p^A_B \beta_{BA} \beta_{BB} \left[ -1 \right] p^A_B \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right] \beta_{BA} \beta_{BB} \left[ -1 \right]

The above equation characterizes the percolation condition of the cooperative secondary network consisting of \( S_N \) and \( SN_B \). In the case without cooperation, \( \beta_{AA} \) and \( \beta_{BA} \) are equal zero, which recovers the percolation condition of a single secondary network \( S_N \) (or \( SN_B \)). That is, (20) reduces to the percolation threshold \( \beta_{AA} = 1/p^A_B \) (or \( \beta_{BB} = 1/p^B_B \)) as described in Section II-C. The above result can be easily extended to the case with cooperation among multiple heterogeneous secondary networks.
In Fig. 4, without cooperation among $SN_A$ and $SN_B$, the mean component size of $SN_A$ ($SN_B$) never blows up since $p_{A}^{B} \beta_{A,A} < 1$ ($p_{B}^{A} \beta_{B,B} < 1$). Note that $\beta_{A,B} = \beta_{B,A} = 0$ when there is no cooperation between $SN_A$ and $SN_B$. This example demonstrates that percolation of the secondary network is facilitated by cooperation.

V. CONCLUSION

We have investigated the connectivity of cooperative secondary network from a percolation-based perspective. First, the degree distribution of an SU is derived considering interference from multiple heterogeneous primary networks under general fading channel model. Second, the active probability of an SU is obtained considering interference temperature constraints at PRs. Third, the relationship between the degree distribution of an SU and the connectivity of the secondary network in percolation sense is established by using the site-percolation model. Fourth, the above steps are extended to the cooperative secondary network, under which each secondary network’s node may have other secondary networks’ nodes acting as relays. The benefits of cooperation among multiple secondary networks are justified in terms of the percolation threshold. Connectivity and energy efficiency of the CRN are facilitated by the proposed cooperation framework.

REFERENCES